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AUTOMORPHISMS OF DISTANCE-REGULAR GRAPH WITH INTERSECTION ARRAY $\{25, 16, 1; 1, 8, 25\}$ [1]

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Abstract: Makhnev and Samoilenko have found parameters of strongly regular graphs with no more than 1000 vertices, which may be neighborhoods of vertices in antipodal distance-regular graph of diameter 3 and with $\lambda = \mu$. They proposed the program of investigation vertex-symmetric antipodal distance-regular graphs of diameter 3 with $\lambda = \mu$, in which neighborhoods of vertices are strongly regular. In this paper we consider neighborhoods of vertices with parameters $(25, 8, 3, 2)$.

Key words: Strongly regular graph, Distance-regular graph.

Introduction

We consider undirected graphs without loops and multiple edges. Given a vertex a in a graph Γ, we denote by $\Gamma_i(a)$ the subgraph induced by Γ on the set of all vertices, that are at the distance i from a. The subgraph $[a] = \Gamma_1(a)$ is called the *neighborhood of the vertex* a. Let $\Gamma(a) = \Gamma_1(a)$, $a^{\perp} = \{a\} \cup \Gamma(a)$. If graph Γ is fixed, then instead of $\Gamma(a)$ we write $[a]$. For the set of vertices X of graph Γ through $X^{\perp}$ denote $\cap_{x \in X} x^{\perp}$.

Let Γ be an antipodal distance-regular graph of diameter 3 and $\lambda = \mu$, in which neighborhoods of vertices are strongly-regular graphs. Then Γ has intersection array $\{k, \mu(r-1), 1; 1, \mu, k\}$, and spectrum $k^1, \sqrt{k}^f, -1^k, -\sqrt{k}^f$, where $f = (k+1)(r-1)/2$. In the case $r = 2$ we obtain Taylor's graph, in which $k' = 2\mu'$. Conversely, for any strongly regular graph with parameters $(v', 2\mu', \lambda', \mu')$ there exists a Taylor's graph, in which neighborhoods of vertices are strongly regular with relevant parameters.

In [1]there were chosen strongly-regular graphs with no more than 1000 vertices, which may be neighborhoods of vertices of antipodal distance-regular graph of diameter 3 and $\lambda = \mu$. There is provided a research program of the study of vertex-symmetric antipodal distance-regular graphs of diameter 3 with $\lambda = \mu$, in which neighborhoods of vertices are strongly regular with parameters from Proposition 1.

Proposition 1. *Let Δ be a strongly-regular graph with parameters (v, k, λ, μ). If $(r-1)k = v-k-1$, $v \leq 1000$ and number $(v+1)(r-1)$ is even, then either $r = 2$, or parameters (v, k, λ, μ, r) belong to the following list:*

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(1) $(16, 5, 0, 2, 3)$, $(25, 8, 3, 2, 3)$, $(49, 12, 5, 2, 4)$, $(64, 21, 8, 6, 3)$, $(81, 16, 7, 2, 5)$, $(81, 20, 1, 6, 4)$, $(85, 14, 3, 2, 6)$, $(99, 14, 1, 2, 7)$, $(100, 33, 8, 12, 3)$, $(121, 20, 9, 2, 6)$, $(121, 30, 11, 6, 4)$, $(121, 40, 15, 12, 3)$, $(126, 25, 8, 4, 5)$, $(133, 44, 15, 14, 3)$, $(169, 24, 11, 2, 7)$, $(169, 42, 5, 12, 4)$, $(169, 56, 15, 20, 3)$, $(176, 25, 0, 4, 7)$, $(196, 39, 14, 6, 5)$, $(196, 65, 24, 20, 3)$;

(2) $(225, 28, 13, 2, 8)$, $(225, 56, 19, 12, 4)$, $(243, 22, 1, 2, 11)$, $(256, 51, 2, 12, 5)$, $(256, 85, 24, 30, 3)$, $(261, 52, 11, 10, 5)$, $(288, 41, 4, 6, 7)$, $(289, 32, 15, 2, 9)$, $(289, 48, 17, 6, 6)$, $(289, 72, 11, 20, 4)$, $(289, 96, 35, 30, 3)$, $(305, 76, 27, 16, 4)$, $(325, 54, 3, 10, 6)$, $(351, 50, 13, 6, 7)$, $(351, 70, 13, 14, 5)$, $(352, 39, 6, 4, 9)$, $(361, 36, 17, 2, 10)$, $(361, 72, 23, 12, 5)$, $(361, 90, 29, 20, 4)$, $(361, 120, 35, 42, 3)$;

(3) $(400, 57, 20, 6, 7)$, $(400, 133, 48, 42, 3)$, $(441, 40, 19, 2, 11)$, $(441, 88, 7, 20, 5)$, $(441, 110, 19, 30, 4)$, $(484, 161, 48, 56, 3)$, $(495, 38, 1, 3, 13)$, $(505, 84, 3, 16, 6)$, $(507, 46, 5, 4, 11)$, $(512, 73, 12, 10, 7)$, $(529, 44, 21, 2, 12)$, $(529, 66, 23, 6, 8)$, $(529, 88, 27, 12, 6)$, $(529, 132, 41, 30, 4)$, $(529, 176, 63, 56, 3)$, $(540, 49, 8, 4, 11)$, $(576, 115, 18, 24, 5)$;

(4) $(625, 48, 23, 2, 13)$, $(625, 156, 29, 42, 4)$, $(625, 208, 63, 72, 3)$, $(640, 71, 6, 8, 9)$, $(649, 72, 15, 7, 9)$, $(649, 216, 63, 76, 3)$, $(676, 75, 26, 6, 9)$, $(676, 135, 14, 30, 5)$, $(704, 37, 0, 2, 19)$, $(729, 52, 25, 2, 14)$, $(729, 104, 31, 12, 7)$, $(729, 182, 55, 42, 4)$, $(736, 105, 20, 14, 7)$, $(768, 59, 10, 4, 13)$, $(784, 261, 80, 90, 3)$;

(5) $(837, 76, 15, 6, 11)$, $(841, 56, 27, 2, 15)$, $(841, 84, 29, 6, 10)$, $(841, 140, 39, 20, 6)$, $(841, 168, 47, 30, 5)$, $(841, 210, 41, 56, 4)$, $(841, 280, 99, 90, 3)$, $(847, 94, 21, 9, 9)$, $(848, 121, 24, 16, 7)$, $(901, 60, 3, 4, 15)$, $(961, 60, 29, 2, 16)$, $(961, 120, 35, 12, 8)$, $(961, 160, 9, 30, 6)$, $(961, 192, 23, 42, 5)$, $(961, 240, 71, 56, 4)$, $(961, 320, 99, 100, 3)$, $(1000, 111, 14, 12, 9)$.

Graphs with local subgraphs having parameters $(64, 21, 8, 6)$, $(81, 16, 7, 2)$, $(85, 14, 3, 2)$ and $(99, 14, 1, 2)$ were investigated in [2], [3], [4] and [5]. In this article we investigate parameters $(25, 8, 3, 2, 3)$, i.e. this graph is locally 5×5-grid. In [6] it is proved that distance-regular locally 5×5-grid of diameter more then 2 is either isomorphic to the Johnson's graph $J(10, 5)$ or has an intersection array $\{25, 16, 1; 1, 8, 25\}$.

Theorem 1. *Let* Γ *be a distance-regular graph with intersection array* $\{25, 16, 1; 1, 8, 25\}$, $G = \mathrm{Aut}(\Gamma)$, *g is an element of prime order p in G and* $\Omega = \mathrm{Fix}(g)$ *contains exactly s vertices in t antipodal classes. Then* $\pi(G) \subseteq \{2, 3, 5, 13\}$ *and one of the following assertions holds:*

(1) Ω *is empty graph and* $p \in \{2, 3, 13\}$;

(2) $p = 5$, $t = 1$, $\alpha_3(g) = 0$, $\alpha_1(g) = 50l + 25$ *and* $\alpha_2(g) = 50 - 50l$;

(3) $p = 3$, $s = 3$, $t = 2, 5, 8$, $\alpha_3(g) = 0$, $\alpha_1(g) = 30l + 16 - 11t$ *and* $\alpha_2(g) = 62 - 30l + 8t$;

(4) $p = 2$, *and either* $s = 1$, Ω *is t-clique*, $t = 2, 4, 6$, $\alpha_3(g) = 2t$, $\alpha_1(g) = 20l - t + 6$ *and* $\alpha_2(g) = 72 - 20l - 2t$, *or* $s = 3$, $t \leq 8$, *t is even*, $\alpha_3(g) = 0$, $\alpha_1(g) = 20l - 11t + 6$ *and* $\alpha_2(g) = 72 - 20l + 8t$.

Corollary 1. *Let* Γ *be a distance-regular graph with intersection array* $\{25, 16, 1; 1, 8, 25\}$ *and a group* $G = \mathrm{Aut}(\Gamma)$ *acts transitively on the set of vertices of* Γ. *Then one of the following assertions holds:*

(1) Γ *is a Cayley graph, G is the a Frobenius group with the kernel of order* 13 *and with the complement of order* 6;

(2) Γ *is a arc-transitive Maton's graph and the socle of G is isomorphic to* $L_2(25)$;

(3) *G is an extension of a group Q of order* 2^{12} *by the group* $T = L_3(3)$, $|Q : Q_{\{F\}}| = 2$, $T_{\{F\}}$ *is an extension of group* E_9 *by* $SL_2(3)$, *T acts irreducibly on Q and for an element f of order* 13 *in G we have* $C_Q(f) = 1$.

1. Proof of the Theorem

Note that there is Delsarte boundary (proposition 4.4.6 from [7]) of maximum order of clique in distance-regular graph with intersection array $\{25, 16, 1; 1, 8, 25\}$ and spectrum $25^1, 5^{26}, -1^{25}, -5^{26}$ no more than $1 - k/\theta_d = 1 + 25/5 = 6$. If C is 6-clique in Γ, then each vertex not in C is adjacent to 0 or to $b_1/(\theta_d + 1) + 1 - k/\theta_d = 2$ vertices in C.

Lemma 1. *Let Γ be a distance-regular graph with intersection array $\{25, 16, 1; 1, 8, 25\}$, $G = \mathrm{Aut}(\Gamma)$ and $g \in G$. If ψ is the monomial representation of a group G in $GL(78, \mathbf{C})$, χ_1 is the character of the representation ψ on subspace of eigenvectors of dimension 26, corresponding to the eigenvalue 5, χ_2 is the character of the representation ψ on subspace of dimension 25, then $\chi_1(g) = (10\alpha_0(g)+2\alpha_1(g)-\alpha_2(g)-5\alpha_3(g))/30$, $\chi_2(g) = (\alpha_0(g)+\alpha_3(g))/3-1$. If $|g| = p$ is prime, then $\chi_1(g) - 26$ and $\chi_2(g) - 25$ are divided by p.*

P r o o f. We have

$$Q = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 26 & 26/5 & -13/5 & -13 \\ 25 & -1 & -1 & 25 \\ 26 & -26/5 & 13/5 & -13 \end{pmatrix}.$$

Therefore $\chi_1(g) = (10\alpha_0(g) + 2\alpha_1(g) - \alpha_2(g) - 5\alpha_3(g))/30$. Substituting $\alpha_2(g) = 78 - \alpha_0(g) - \alpha_1(g) - \alpha_3(g)$, we obtain $\chi_1(g) = (11\alpha_0(g) + 3\alpha_1(g) - 4\alpha_3(g))/30 - 13/5$.

Similarly, $\chi_2(g) = (25\alpha_0(g) - \alpha_1(g) - \alpha_2(g) + 25\alpha_3(g))/78$. Substituting $\alpha_1(g) + \alpha_2(g) = 78 - \alpha_0(g) - \alpha_3(g)$, we obtain $\chi_2(g) = (\alpha_0(g) + \alpha_3(g))/3 - 1$.

The remaining assertions follow from Lemma 1 in [8]. The proof is complete. □

Let further in the paper Γ be a distance-regular graph with intersection array $\{25, 16, 1; 1, 8, 25\}$, $G = \mathrm{Aut}(\Gamma)$, g is an element of prime order p in G and $\Omega = \mathrm{Fix}(g)$.

Lemma 2. *If Ω is an empty graph, then either $p = 13$, $\alpha_1(g) = 26$ and $\alpha_2(g) = 52$, or $p = 3$, $\alpha_3(g) = 9s+6$, $s < 8$, $\alpha_1(g) = 54+12s-30l$ and $\alpha_2(g) = 18-21s+30l$, $l \le 5$, or $p = 2$, $\alpha_3(g) = 0$, $\alpha_1(g) = 20l + 6$ and $\alpha_2(g) = 72 - 20l$, $l \le 3$.*

P r o o f. Let Ω be an empty graph and $\alpha_i(g) = pw_i$ for $i > 0$. Since $v = 78$, we have $p \in \{2, 3, 13\}$.

Let $p = 13$. Then $\alpha_3(g) = 0$, $\alpha_1(g) + \alpha_2(g) = 78$ and $\chi_1(g) = (2\alpha_1(g) - \alpha_2(g))/30 = 13(w_1 - 2)/10$. This implies $\alpha_1(g) = 26$ and $\alpha_2(g) = 52$.

Let $p = 3$. Then $\chi_2(g)-25 = \alpha_3(g)/3-26$ is divided by 3, $\alpha_3(g) = 9s+6$, $s \le 8$ and $\alpha_2(g) = 72-9s-\alpha_1(g)$. Furthermore, the number $\chi_1(g) = (2\alpha_1(g)-\alpha_2(g)-45s-30)/30 = (3w_1-12s-34)/10$ is congruent to 2 modulo 3. This implies $\alpha_1(g) = 54 + 12s - 30l$ and $\alpha_2(g) = 18 - 21s + 30l$, $l \le 5$. In case $s = 8$ we have $\alpha_3(g) = 78$ and $\langle g \rangle$ acts regularly on each antipodal class. By lemma 4 in [9] 3 must divide $k + 1 = 26$, we have a contradiction.

Let $p = 2$. Then $\alpha_3(g) = 0$, $\alpha_1(g) + \alpha_2(g) = 78$, the number $\chi_1(g) = (\alpha_1(g) - 26)/10$ is even, $\alpha_1(g) = 20l + 6$ and $\alpha_2(g) = 72 - 20l$, $l \le 3$. □

In Lemmas 3–6 it is assumed that there are t antipodal classes intersecting the Ω on s vertices. Then p divides $26 - t$ and $3 - s$. Let F be an antipodal class, containing the vertex $a \in \Omega$, $F \cap \Omega = \{a, a_2, ..., a_s\}$, $b \in \Omega(a)$. By $F(x)$ we denote an antipodal class containing vertex x.

Lemma 3. *The following assertions hold:*
(1) *if* $t=1$, *then* $p=5$, $\alpha_3(g)=0$, $\alpha_1(g)=50l+25$ *and* $\alpha_2(g)=50-50l$;
(2) *if* p *more than* 3, *then* $p=5$ *and* $t=1$;
(3) *if* $s=1$, *then* $p=2$, $t=2,4,6$, $\alpha_3(g)=2t$, $\alpha_1(g)=20l-t+6$ *and* $\alpha_2(g)=72-20l-2t$.

P r o o f. If $s=3$, then each vertex from $\Gamma-\Omega$ is adjacent to t vertices in Ω, so $t\le 8$.

Let $t=1$. As p divides $26-t$, then $p=5$, $s=3$, $\alpha_2(g)=75-\alpha_1(g)$, the number $\chi_1(g)=(\alpha_1(g)-15)/10$ is congruent to 1 modulo 5. This implies $\alpha_1(g)=50l+25$.

Let $p>3$, $\alpha_1(g)=pw_1$. Then $s=3$, $|\Omega|=3t$, Ω is a regular graph by degree $t-1$ and p divides $26-t$.

If $p>7$, then Ω is a distance-regular graph with intersection array $\{t-1,16,1;1,8,t-1\}$, we come to a contradiction.

Let $p=7$. As p divides $26-t$, then $t=5$, the subgraph $\Omega(b)$ contains 2 vertices in $a^\perp$ and a vertex from $[a_2]$ and from $[a_3]$, so Ω is a distance-regular graph with intersection array $\{4,1,1;1,1,4\}$, it is a contradiction with the fact that $r=3$.

Let $p=5$. As p divides $26-t$, then $t=1,6$. If $t=6$, then the subgraph $\Omega(b)$ contains a vertex in $a^\perp$, 3 vertices from $[a_2]$ and 3 vertices from $[a_3]$, we come to a contradiction.

Let $s=1$. Then $p=2$, $t\le 6$, $\alpha_3(g)=2t$, $\alpha_2(g)=78-\alpha_1(g)-3t$, and $\chi_1(g)=(\alpha_1(g)+t-26)/10$ is even. This implies that $\alpha_1(g)=20l-t+6$. □

Lemma 4. *If* $p=3$, *then* $s=3$, $t=2,5,8$, $\alpha_3(g)=0$, $\alpha_1(g)=30l+16-11t$ *and* $\alpha_2(g)=62-30l+8t$.

P r o o f. Let $p=3$. Then $s=3$, $t=2,5,8$, $\alpha_2(g)=78-\alpha_1(g)-3t$, and the number $\chi_1(g)=(11t+\alpha_1(g)-26)/10$ is congruent to 2 modulo 3. This implies that $\alpha_1(g)=30l+16-11t$. In the case $t=2$ graph Ω is a union of 3 isolated edges. □

Lemma 5. *If* $p=2$, $s=3$, *then* t *is even*, $t\le 8$, $\alpha_3(g)=0$, $\alpha_1(g)=20l-11t+6$ *and* $\alpha_2(g)=72-20l+8t$.

P r o o f. Let $p=2$, $s=3$. Then t is even, $t\le 8$, $\alpha_3(g)=0$, $\alpha_2(g)=78-3t-\alpha_1(g)$.
The number $\chi_1(g)=(11t+\alpha_1(g)-26)/10$ is even, so $\alpha_1(g)=20l-11t+6$. □

Lemmas 2–5 imply the proof of the Theorem.

2. Proof of Corollary

Let the group G acts transitively on the set of vertices of the graph Γ. Then for a vertex $a\in\Gamma$ subgroup $H=G_a$ has index 78 in G. By Theorem we have $\{2,3,13\}\subseteq\pi(G)\subseteq\{2,3,5,13\}$.

Lemma 6. *Let* f *be an element of order* 13 *in* G. *Then* $\mathrm{Fix}(f)$ *is an empty graph*, $\alpha_1(f)=26$ *and the following assertions hold:*
(1) *if* g *is an element of prime order* $p\ne 13$ *in* $C_G(f)$, *then* $p=2$, Ω *is an empty graph*, $\alpha_1(g)=26$ *and* $|C_G(f)|$ *is not divided by* 4;
(2) *either* $|G|=78$ *or* $F(G)=O_2(G)$;
(3) *if* G *is nonsolvable group, then the socle* $\bar T$ *of the group* $\bar G=G/F(G)$ *is isomorphic to* $L_2(25)$, $L_3(3)$, $U_3(4)$, $L_4(3)$ *or* ${}^2F_4(2)'$.

P r o o f. By Lemma 2 Fix(f) is an empty graph and $\alpha_1(f) = 26$.

Suppose that g is an element of prime order $p \neq 13$ in $C_G(f)$. As f acts without fixed points on Ω then by Theorem Ω is an empty graph, $p = 2$ and $\alpha_1(g) = 20l + 6$ divided by 13. This implies that $\alpha_1(g) = 26$ and $|C_G(f)|$ is not divided by 4.

Let $Q = O_p(G) \neq 1$. If $p = 13$, then $|G|$ divides $26 \cdot 12$. In this case $C_G(f) = \langle f \rangle$, otherwise for an involution g of $C_G(f)$ we obtain a contradiction with the action of element of order 3 of G on $\{u \mid d(u, u^g) = 1\}$. Let the involution g inverts f, h is an element of order 3 in $C_G(g)$. From action h on $\{u \mid d(u, u^g) = 1\}$ it follows that $\alpha_1(g) = 20l + 6$ is divided by 3. In each case $\alpha_1(g)$ is not divided by 4 and $|G| = 78$.

If $p = 3$, then Q fixes some antipodal class. This implies that Q fixes each antipodal class. By Lemma 3 in [9] G does not contain subgroups of order 3, which are regular on each antipodal class, we come to a contradiction. So, if $|G| \neq 78$ we have $F(G) = O_2(G)$.

Let $\bar{T}$ be the socle of the group $\bar{G} = G/F(G)$. Note that 13 divides $|\bar{T}|$ and by Theorem 1 in [10] group $\bar{T}$ is isomorphic to $L_2(25)$, $L_3(3)$, $U_3(4)$, $L_4(3)$, ${}^2F_4(2)'$. □

Let us to prove the Corollary. As $\bar{T}$ contains a subgroup of index dividing 26, then the group $\bar{T}$ is isomorphic to $L_2(25)$ (and $\bar{T}_{\{F\}}$ is the extension of a group of order 25 by group of order 12) or $L_3(3)$ (and $\bar{T}_{\{F\}}$ is the extension of a group of order 9 by $SL(2,3)$).

In the first case $F(G)$ fixes each antipodal class and $F(G) = 1$. This implies that Γ is the arc-transitiv Maton's graph.

In the second case for $Q = F(G)$ we have $|Q : Q_{\{F\}}| = 2$ and $\bar{T}$ acts irreducibly on Q. Further, for the element f of order 13 of G by Lemma 6 the number $|C_Q(f)|$ divides 2. As Q is either 12-dimensional module over F_2, or 16-dimensional module over F_{16}, or 26-dimensional module over F_2, then $|Q| = 2^{12}$ and $C_Q(f) = 1$. The Corollary is proved.

3. Conclusion

We found possible automorphisms of a distance regular graph with intersection array $\{25, 16, 1; 1, 8, 25\}$. This completes the research program of vertex-symmetric antipodal distance-regular graphs of diameter 3 with $\lambda = \mu$, in which neighborhoods of vertices are strongly regular with parameters from Proposition 1.

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