SIMPLIFIED MODEL OF THE HEAT EXCHANGE PROCESS IN ROTARY REGENERATIVE AIR PRE–HEATER[1]

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Abstract: A simplified mathematical model of a rotary regenerative air pre-heater (RRAP) is suggested and studied based on the averaged dynamics of the heat exchange process between nozzles and a heat carrier (i.e. air or gas-smoke mixture). Averaging in both spatial coordinates and time gives a linear discrete system that allows deriving explicit formulas for determining the characteristics of the air heater and establishing some properties such as periodicity, stability, ergodicity and others.

Key words: Heat exchange, Cyclic process, Averaging, Linear discrete system, Stability, Ergodicity, Inverse problem.

Introduction

A rotary regenerative air pre-heater (RRAP) is a special unit, usually applied in thermal power plants (TPP) in order to increase its efficiency by heating the air which is blowing into a boiler of the plant by means of an exhaust of hot mixture of smoke and gas (from now on simply gas) generated in fuel combustion. The RRAP essentially reduces the thermal pollution of atmosphere [6], [8–12].

Currently, several types of RRAPs are used in TPP. In the present paper, we consider the case of a unit with the main part consisting of a rotating cylindrical drum with metal nozzles of high thermal conductivity. The region of space occupied by the wheel of the RRAP is divided into two parts B_A and B_G by a fixed conditional plane passing through the axis of the cylinder. During the work of the RRAP, the atmospheric air passes through the part B_A in one direction parallel to the axis of the drum, being heated by the nozzles; as a result, the temperature of the nozzles is reduced. Through the part B_G, the gas flows in the opposite direction, being cooled by the heat output to the nozzles. The final transfer of heat from the hot gas to the cool air occurs due to the rotation of the drum around its axis.

Monitoring and control of the temperatures of air and gas leaving the RRAP and especially the temperature of the nozzles is the important problem for the effective exploitation of the RRAP [3], [15], [17–19]. The direct measurement of the temperatures of air and gas of both inlet and outlet is easily carried out; in contrast, the observation and control of the nozzle temperature require to use sophisticated measuring tools and represent an important problem. Therefore, the treatment for mathematical modeling of the heat exchange process in the RRAP is reasonable.

In recent decades, a variety of mathematical models of the heat exchange process in RRAPs were suggested [1–3], [5] and see also [8]. It should be noted that the thermodynamic process in RRAP nozzles can be in principle described by the classical heat conduction equation, but, without additional simplifying assumptions, it is difficult to write out the boundary conditions since the nozzle system has a rather complex geometry. On the other hand, modeling of the thermodynamic process of air heating and gas cooling passing through the drum of the RRAP is even more challenging problem pertaining to thermos-aerodynamics [8]. Furthermore, the necessity of considering the turbulence of flow that arises when air and gas pass through the RRAP drum also adds complexity. Because of these and other features, all mathematical models of the RRAP are built under essential simplifying assumptions.

In the present paper, we propose a mathematical model of the thermodynamic process in the RRAP based on averaging the quantities associated with the heat exchange process between the nozzles, air and gas in both spatial coordinates and time interval. As a result, we obtain rather simple linear discrete equation. This allows us to write out explicit calculating formulas for the current values of parameters, and find steady and periodical states, establish the ergodicity and other properties. Then, we provide appropriate calculations for the case when averaging over the time variable is carried out on a period of time equal to a half-cycle of rotation of the RRAP drum. In subsequent parts of the work, models with averaging performed over a small time interval are considered, a comparative analysis of numerical results and experimental data is fulfilled.

1. Derivation of equations

Let the RRAP drum be of the form of a cylinder $x^2 + y^2 \leq R^2$, $0 \leq z \leq H$. Assume that the parts B_A and B_G are described by conditions $y \geq 0$ and $y \leq 0$, respectively. Let $\Theta(t, x, y, z)$ denote the temperature at the point (x, y, z) of the drum occupied by the nozzles at time t, $t \geq 0$, and let $T(t, x, y, z)$ denote the temperature of heat transfer (air or gas) at the point (x, y, z) of the drum outside the nozzles. The pair of quantities $\Theta(t, x, y, z)$ and $T(t, x, y, z)$ completely characterizes the heat change process in the RRAP drum. However, as mentioned above, due to the complexity of the configuration of the nozzles, the initial-boundary problem for corresponding system of equations of thermo-aerodynamics is too hard to be investigated by analytical methods. One of the ways to overcome such complexity is the method of averaging [4], [7]. To this end, we consider the corresponding average values

$$\Theta_A(t) = \frac{1}{V(B_A^\bullet)} \int_{B_A^\bullet} \Theta_A(t, x, y, z) dv, \quad \Theta_G(t) = \frac{1}{V(B_G^\bullet)} \int_{B_G^\bullet} \Theta_G(t, x, y, z) dv,$$

$$T_A(t) = \frac{1}{V(B_A^\circ)} \int_{B_A^\circ} T_A(t, x, y, z) dv, \quad T_G(t) = \frac{1}{V(B_G^\circ)} \int_{B_G^\circ} T_G(t, x, y, z) dv,$$

where $B_A^\bullet = B_A \cap \bar{B}$, $B_G^\bullet = B_G \cap \bar{B}$, $\bar{B}$ is the part of the drum occupied by the nozzles, $B_A^\circ = B_A \backslash B_A^\bullet$, $B_G^\circ = B_G \backslash B_G^\bullet$, dv is the volume element, V denotes the volume of the corresponding part of the drum.

Next, we perform averaging over the time intervals $I(n) = [nh, \ (n+1)h]$ as well, where h is the half-turn time of the RRAP drum, $n = 0, 1, 2, \ldots$. Denote the average temperatures obtained in this way by

$$x_n = \frac{1}{h} \int_{I(n)} \Theta_A(t) dt, \quad y_n = \frac{1}{h} \int_{I(n)} \Theta_G(t) dt,$$

$$u_n = \frac{1}{h} \int_{I(n)} T_A(t) dt, \quad v_n = \frac{1}{h} \int_{I(n)} T_G(t) dt.$$

Equations connecting these quantities are derived under the following simplifying assumptions:

1. for $nh \leq t < (n+1)h$, the RRAP drum is motionless, the portion of air (respectively, gas) filling the part B_A° (respectively, B_G°) is also motionless, the heat exchange between the heat carrier and nozzles occurs in accordance with the linear Newton law [8];

2. at the time $t = (n+1)h$, the heated portion of air with temperature u_n leaves the area B_A°, the gas portion cooled down to temperature v_n leaves the region B_G°, the drum turns to 180° with jump (i.e., the part $B_A^{\bullet}$ goes to the part $B_G^{\bullet}$ and vice versa); then the part B_A° is filled in by a new portion of air from outward (or from calorifer in case of its connection to the RRAP, [12]) of temperature p_n, and the part B_G° is filled in by a new portion of gas of temperature q_n.

In accordance with the Newton law, we have the following relations:

$$x_{n+1} = (1 - \beta h)y_n + \beta h q_n, \quad y_{n+1} = (1 - \alpha h)x_n + \alpha h p_n, \tag{1.1}$$

$$u_n = p_n + \gamma h(x_n - p_n), \quad v_n = q_n + \delta h(y_n - q_n), \tag{1.2}$$

where α, β, γ, δ are parameters depending on characteristics of the heat exchange process in the RRAP (the geometry, the heat capacity of the carcass of drum, the system of nozzles and their structure, the humidity of air and gas and their thermodynamic characteristics, the coefficients of heat conductivity and diffusion, parameters that characterize the heat exchange process on the contact surface of the nozzles with air and gas and etc.).

Relations (1.1) and (1.2) have been obtained as a result of extremely simplifying assumptions on the heat exchange process in the RRAP. Nevertheless, due to such simplification, system (1.1) allows a fairly complete analysis; therefore, it can serve as a basic model to describe the work of the RRAP.

2. Solution of the system

Introducing the vectors $z_n = (x_n, y_n)^T$ and $r_n = h(\beta q_n, \alpha p_n)^T$, where T is the transpose sign to transform the row-vector to the column-vector, and the matrix

$$A = \begin{pmatrix} 0 & 1 - \beta h \\ 1 - \alpha h & 0 \end{pmatrix},$$

one can rewrite system (1.1) as follows:

$$z_{n+1} = Az_n + r_n, \quad n = 0, 1, 2, \ldots. \tag{2.1}$$

All further reasoning is conducted under the assumption $0 < \alpha h$, $\beta h < 1$, called the physical realizability of the model. This condition implies the fact that the eigenvalues of the matrix A, which are $\pm\mu$ where $\mu = \sqrt{(1 - \alpha h)(1 - \beta h)}$, belong to the interval $(-1, \ 1)$ and therefore all the solutions of system (1.1) are asymptotically stable [13,14], [16].

The solution of equation (2.1) can be written by the Cauchy formula [13]

$$z_n = A^n z_0 + \sum_{k=0}^{n-1} A^{n-1-k} r_k. \tag{2.2}$$

Since $A^n = \mu^n E$ for even numbers n, and $A^n = \mu^{n-1} A$ for odd numbers n, then equation (2.2) can be transformed to the form

$$z_n = \mu^n z_0 + A \sum_{j=0}^{n/2-1} \mu^{n-2-2j} r_{2j} + \sum_{j=0}^{n/2-1} \mu^{n-2-2j} r_{2j+1},$$

for even n, and to the form

$$z_n = \mu^{n-1} A z_0 + A \sum_{j=1}^{(n-1)/2} \mu^{n-1-2j} r_{2j-1} + \sum_{j=0}^{(n-1)/2} \mu^{n-1-2j} r_{2j}$$

for odd n.

Based on these relations, one can easily obtain explicit formulas for x_n and y_n. In practice, however, it is more convenient to calculate them, in case of need, directly from equations (1.1) that are accommodated to computer calculations. In the rest of the paper, some properties of solution of systems (1.1), (1.2) are established.

3. Steady state and periodic regimes

Let us first consider the case of a steady state, which occurs under the assumption of constant incoming flows. Such a state can be established in the periods of time measured in hours, if the parameters of the air and the energy load at the TPP remain virtually unchanged. Since h is measured in minutes, the work of the RRAP actually consists of long periods of steady states and relatively short transition intervals from one steady state to another. Therefore, it is important to determine the parameters of the RRAP under steady state conditions.

Thus, let

$$p_n \equiv \bar{p}, \quad q_n \equiv \bar{q}, \quad n = 0, 1, 2, \ldots, \quad \bar{r} = (\beta \bar{q}, \quad \alpha \bar{p})^T.$$

Then formula (2.2) can be simplified even more: for even n

$$z_n = \mu^n z_0 + (1 - \mu^n)(E - A)^{-1} \bar{r}, \tag{3.1}$$

or in the component form

$$x_n = \mu^n x_0 + \frac{1 - \mu^n}{\alpha + \beta - \alpha\beta h}[p + q(1 - \beta h)],$$

$$y_n = \mu^n y_0 + \frac{1 - \mu^n}{\alpha + \beta - \alpha\beta h}[q + p(1 - \alpha h)],$$

and for odd n

$$z_n = \mu^{n-1} A z_0 + \frac{1}{\alpha + \beta - \alpha\beta h}(E - \mu^n A)(E - A)^{-1} r,$$

or

$$x_n = \mu^{n-1}(1 - \beta h) y_0 + \frac{1}{\alpha + \beta - \alpha\beta h}[p(1 - \mu^{n+2}) + q(1 - \beta h)(1 - \mu^{n-1})],$$

$$y_n = \mu^{n-1}(1 - \alpha h) x_0 + \frac{1}{\alpha + \beta - \alpha\beta h}[q(1 - \mu^{n+2}) + p(1 - \alpha h)(1 - \mu^{n-1})].$$

For the steady state, we obtain $\bar{z} = (E - A)^{-1}\bar{r}$ from the equation $\bar{z} = A\bar{z} + \bar{r}$ (the invertibility of the matrix $E - A$ follows from the condition of physical realizability) or in the component form

$$\bar{x} = \frac{1}{\alpha + \beta - \alpha\beta h}(\alpha p + \beta q(1 - \beta h)),$$

$$\bar{y} = \frac{1}{\alpha + \beta - \alpha\beta h}(\beta q + \alpha p(1 - \alpha h)). \tag{3.2}$$

Due to the fact noticed above, steady state (3.2) is asymptotically stable. On time intervals measured in weeks, a periodic state for the RRAP can be formed.

Theorem 1. *Let the sequence r_n be periodic with the period m, $m \geq 2$, i.e. $r_{n+m} = r_n$, where $n = n_0$, $n_0 + 1$, $n_0 + 2$, $\ldots$; n_0 is some fixed value of n. Then system (2.1) has a unique periodic solution of period m, which is asymptotically stable.*

P r o o f. The uniqueness and asymptotic stability follows from the condition of physical realizability. To establish the existence, we set

$$z_n^* = (E - A^m)^{-1} \sum_{k=0}^{m-1} A^{m-1-k} r_k \qquad (3.3)$$

(it follows from the condition of physical realizability that the matrix $E - A^m$ is invertible).

Let z_n^* be a trajectory with the initial point z_0^*. Rewrite relation (3.3) as follows:

$$z_n^* = A^m z_0^* + \sum_{k=0}^{m-1} A^{m-1-k} r_k = z_m^*.$$

Then,

$$z_{n+m}^* = A^{n+m} z_0^* + \sum_{k=0}^{m-1} A^{n+m-1-k} r_k + \sum_{k=m}^{n+m-1} A^{n+m-1-k} r_k. \qquad (3.4)$$

The sum of the first two summands in the right hand side of (3.4) is $A^n z_m^*$, the third one is equal to

$$\sum_{k=0}^{n-1} A^{n-1-k} r_{k+m} = \sum_{k=0}^{n-1} A^{n-1-k} r_k$$

since r_k is periodic. Thus,

$$\sum_{k=0}^{n-1} A^{n-1-k} r_k = z_n - A^n z_0.$$

Now, in accordance with (3.4), we obtain

$$z_{n+m}^* = A^n z_m^* + z_n - A^n z_0^* = z_n,$$

and the proof is complete. □

4. Boundedness and ergodicity of the solutions

In time intervals of longer duration, in a changeable external environment and energy load at the TPP, the sequence r_n is not periodic, a fortiori, it is not stationary. In this regard, we give two properties of the solution for more general classes of systems (2.1), which models more or less irregular regime of the heat exchange process.

Theorem 2. *If r_n is a bounded sequence, then each solution of equation (2.1) is also bounded.*

This statement is also a special case of more general theorem [11].

Theorem 3. *Let $\lim\limits_{n \to \infty} r_n = l$. Then each solution z_n approaches the limit $(E - A)^{-1} l$ as $n \to \infty$ independently of z_0.*

P r o o f. First assume that $l = 0$. Let $M = \max_n |r_n|$. Given any $\varepsilon > 0$, choose $n_0(\varepsilon)$ such that $|r_n| < \frac{1}{2}(1 - \|A\|)\varepsilon$ at $n \geq n_0(\varepsilon)$. Then choose $N(n_0(\varepsilon))$ such that

$$\mu^n < \frac{1 - \|A\|}{2M\left(1 - \|A^{n_0}\|\right)}\varepsilon$$

for all $n \geq N(n_0(\varepsilon))$. Clearly, $N(n_0(\varepsilon))$ can be chosen to satisfy $N(n_0(\varepsilon)) \geq n_0(\varepsilon)$. Then, for $n \geq N(n_0(\varepsilon))$, we have

$$|z_n| < \|A^n\| \, |z_0| + \left| \sum_{k=0}^{n_0-1} A^{n-1-k} r_k \right| + \left| \sum_{k=n_0}^{n-1} A^{n-1-k} r_k \right|.$$

Since $0 < \alpha h, \ \beta h < 1$, the first summand tends to zero as $n \to \infty$. Denote two other terms by S_n^I and S_n^{II}. Then, in view of $\|A\| = \mu \in (0, 1)$, we get

$$S_n^I < \|A^{n-n_0}\| \sum_{k=0}^{n_0-1} \|A^{n_0-1-k}\| \, |r_k| < M \|A^{n-n_0}\| \sum_{k=0}^{n_0-1} \|A^{n_0-1-k}\| < M\mu^n \frac{1 - \|A^{n_0}\|}{1 - \|A\|} < \frac{\varepsilon}{2},$$

$$S_n^{II} < \sum_{k=n_0}^{n-1} \|A^{n-1-k}\| \, |r_k| < \frac{1 - \|A\|}{2}\varepsilon \sum_{k=0}^{\infty} \|A^k\| = \frac{1 - \|A\|}{2}\varepsilon\frac{1}{1 - \|A\|} < \frac{\varepsilon}{2}.$$

Hence, $|z_n| \leq S_n^I + S_n^{II} < \varepsilon$ at $n \geq N(n_0(\varepsilon))$, i.e. $z_n \to 0$.

Let now $r_n \to l$ as $n \to \infty$, where $l \neq 0$. We make the change of variables

$$z_n = \bar{z}_n + (E - A)^{-1}l, \quad \bar{r}_n = r_n - l.$$

Then

$$\bar{z}_{n+1} + (E - A)^{-1}l = A\left[\bar{z}_n + (E - A)^{-1}l\right] + \bar{r}_n + l = A\bar{z}_n + \bar{r}_n,$$

where $\bar{r}_n \to 0$. As proved, $\bar{z}_n \to 0$. Therefore, $z_n \to (E - A)^{-1}l$.   □

In general, the change of the parameters characterizing the state of air, as well as the variation of the load at the TPP is random with a hard-determinable distribution function.

Considering the work of the RRAP as a stochastic process, we leave it for the next part of the paper, and we now present another property, taking into account irregular characters of values of phase variables of system (1.1), (1.2).

A sequence a_n is called almost-periodic, if it can be represented in the form $b_n + c_n$, where b_n is periodic and $c_n \to 0$ as $n \to \infty$.

Corollary 1. *If r_n is almost-periodic, then each solution of (2.2) is also almost-periodic.*

Definition. A sequence x_n is called ergodic, if the sequence of Cesaro means

$$\sigma_n = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

converges as $n \to \infty$.

Theorem 4. *If the sequence r_n is ergodic, namely*

$$\rho_n = \frac{r_0 + r_1 + \cdots + r_{n-1}}{n} \to l,$$

then each solution z_n is also ergodic with

$$S_n = \frac{z_1 + z_2 + \cdots + z_n}{n} \to (E - A)^{-1}l.$$

P r o o f. Express S_N as the sum of two terms: $S_N = \Sigma_I + \Sigma_{II}$, where

$$\Sigma_I = \frac{1}{N} \sum_{n=1}^{N} A^n z_0, \quad \Sigma_{II} = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=0}^{n-1} A^{n-1-k} r_k.$$

Then

$$\Sigma_I = \frac{1}{N} \sum_{n=1}^{N} A^n z_0 = \frac{1}{N} A (E-A)^{-1} (E - A^N) z_0 \to 0$$

as $n \to \infty$.

For Σ_{II}, we have

$$\Sigma_{II} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=k+1}^{N} A^{n-1-k} r_k = \frac{1}{N} \sum_{k=0}^{N-1} \left(\sum_{n=0}^{N-1-k} A^n \right) r_k$$

$$= \frac{(E-A)^{-1}}{N} \left[\sum_{k=0}^{N-1} r_k - \sum_{k=0}^{N-1} A^{N-1-k} r_k \right]$$

$$= (E-A)^{-1} \rho_n - \frac{(E-A)^{-1}}{N} \sum_{k=0}^{N-1} A^{N-1-k} r_k.$$

By hypothesis of the theorem, $(E-A)^{-1} \rho_n \to (E-A)^{-1} l$ as $N \to \infty$. Assuming $\rho_k = \xi_k + l$, where $\xi_k \to 0$, we arrive at the equation

$$r_k = (k+1)\rho_{k+1} - k\rho_k = (k+1)\xi_{k+1} - k\xi_k + l.$$

Thus,

$$\frac{(E-A)^{-1}}{N} \sum_{k=0}^{N-1} A^{N-1-k} r_k = \frac{(E-A)^{-1}}{N} \left[\sum_{k=0}^{N-1} A^{N-1-k} [(k+1)\xi_{k+1} - k\xi_k] - \sum_{k=0}^{N-1} A^{N-1-k} l \right].$$

Obviously,

$$\sum_{k=0}^{N-1} A^{N-1-k} l \to 0$$

as $N \to \infty$. Let

$$S = \frac{1}{N} \sum_{k=0}^{N-1} A^{N-k-1} k \xi_k.$$

Show that $S \to 0$ as $N \to \infty$. Indeed,

$$|S| \leq \sum_{k=0}^{N-1} \mu^{N-1-k} \frac{k}{N} |\xi_k| \leq \sum_{k=0}^{N-1} \mu^{N-1-k} |\xi_k|.$$

By Theorem 3, the right-hand side of this inequality tends to 0 as $N \to \infty$. Thus, finally we obtain

$$S_N = \frac{1}{N} \sum_{n=1}^{N} z_n \to (E-A)^{-1} l,$$

which is our claim. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ □

5. Finding the coefficients of the system

As mentioned above, besides the temperature and velocity of incoming air and gas, the drum rotation speed, the heat exchange process in the RRAP depends on many parameters expressing the thermodynamic characteristics of air and gas, the material and geometry of the nozzles of the RRAP, thermal properties of the drum casing etc.

There are a number of works devoted to the mathematical modeling of the RRAP where formulas are given to calculate the values of the parameters above on the basis of molecular physics laws [1–3], [8], [9]. In addition, finding the numerical parameters characterizing the RRAP is only possible with a certain accuracy. From this point of view, it is much easier to define them with a satisfactory accuracy by solving the inverse problem for system (1.1), (1.2) on the basis of empirical data. With regard to model (1.1), (1.2), such a problem consists in calculating the values of α, β, γ, and δ based on the results of measurements of temperatures of outgoing air u_n and gas v_n. Here, there is a wide field of application of the least squares method and tools of mathematical statistics. Here, we confine ourselves to the simplest case, when α, β, γ, and δ are found by measuring u_1, u_2, v_1, and v_2 assuming that incoming streams $p_n = p$, $q_n = q$ are stationary. We can assume that $u_0 = p$, $v_0 = q$, and the values of u_1, u_2, v_1, and v_2 are found by the direct measurements. As a result, we arrive at the problem of finding the unknowns α, β, γ, δ based on the given values h, p, q, u_1, u_2, v_1, v_2, without measuring the values x_n, y_n (the nozzle temperature).

We have

$$x_2 = (1 - \beta h)y_1 + \beta h q, \quad y_2 = (1 - \alpha h)x_1 + \alpha h p, \tag{5.1}$$

$$u_1 = p + \gamma h(x_1 - p), \quad v_1 = q + \delta h(y_1 - q), \tag{5.2}$$

$$u_2 = p + \gamma h(x_2 - p), \quad v_2 = q + \delta h(y_2 - q). \tag{5.3}$$

Set $\bar{u}_k = u_k - p$, $\bar{v}_k = v_k - q$, $k = 1, 2$, (these values have the clear physical meaning). Substituting the values of x_2 and y_2 from (5.1) into (5.3), we obtain the following system

$$\gamma h(x_1 - p) = \bar{u}_1, \quad \delta h(y_1 - q) = \bar{v}_1, \tag{5.4}$$

$$\gamma h[(1 - \beta h)y_1 + \beta h q - p] = \bar{u}_2, \quad \delta h[(1 - \alpha h)x_1 + \alpha h p - q] = \bar{v}_2, \tag{5.5}$$

with 6 unknowns α, β, γ, δ, x_1, y_1. It is nonlinear and, in general, cannot be solved explicitly.

Therefore, we use the fact that, for the values of α, β, γ, δ, there is a priori estimate $0.1 \div 0.6$ and $h < 1$. This allows us to neglect the terms containing $\beta\gamma h^2$ and $\alpha\delta h^2$.

Then equations (5.5) take the form $\gamma h(y_1 - p) = \bar{u}_2$, $\delta h(x_1 - q) = \bar{v}_2$. As a result, for the intermediate unknowns x_1, y_1, we obtain the linear system

$$\frac{x_1 - p}{y_1 - p} = \frac{\bar{u}_1}{\bar{u}_2}, \quad \frac{y_1 - q}{x_1 - q} = \frac{\bar{v}_2}{\bar{v}_1}, \tag{5.6}$$

with the determinant equal to $\bar{u}_1\bar{v}_1 - \bar{u}_2\bar{v}_2$. We call the quantity $\chi = |\bar{u}_1\bar{v}_1 - \bar{u}_2\bar{v}_2|$ the divergence coefficient of the RRAP. The deviation of χ from zero is a characteristic of the RRAP that expresses how the rates of air heating and gas cooling differ. Further, we assume $\chi \neq 0$. It follows from (5.6) that

$$x_1 = \frac{(\bar{u}_1 - \bar{u}_2)\bar{v}_2 p + (\bar{v}_1 - \bar{v}_2)\bar{u}_1 q}{\bar{u}_1\bar{v}_1 - \bar{u}_2\bar{v}_2}, \quad y_1 = \frac{(\bar{u}_1 - \bar{u}_2)\bar{v}_1 p + (\bar{v}_1 - \bar{v}_2)\bar{u}_2 q}{\bar{u}_1\bar{v}_1 - \bar{u}_2\bar{v}_2}.$$

Substituting the values x_1, y_1 into (5.4), (5.5), we obtain the final formulas

$$\alpha h = \frac{x_1 - q}{x_1 - p}\bar{v}_1 - \frac{y_1 - q}{x_1 - p}\bar{v}_2, \quad \beta h = \frac{y_1 - p}{y_1 - q}\bar{u}_1 - \frac{x_1 - p}{y_1 - q}\bar{u}_2.$$

$$\gamma h = \frac{\bar{u}_1}{x_1 - p}, \quad \delta h = \frac{\bar{v}_1}{y_1 - q}.$$

Table 1 shows posteriori values of parameters α, β, γ, δ calculated by these formulas under the assumption that $p = 32°$, $q = 282°$, $h = 0.25$ [4, 5].

$\bar{u}_1$	$\bar{v}_1$	$\bar{u}_2$	$\bar{v}_1$	α	β	γ	δ
3.61	−2.58	6.75	−4.82	0.048	0.068	0.041	0.029

Table 1. Posteriori values of parameters α, β, γ, δ.

6. Conclusion

In the present paper, we have proposed the mathematical model of thermodynamic process of the RRAP, which is described by linear discrete equations. To obtain this, we have used averaging the quantities associated with the heat exchange process between the nozzles, air and gas in both the spatial coordinates and time interval. We have found steady and periodical states, established the ergodicity and other properties. Next, we have studied the cases when the time averaging is performed over the period of time equal to the half-cycle of rotation of the RRAP drum as well as when the time averaging is performed over a small time interval. Finally, we have provided the comparative analysis of the numerical results obtained and the experimental data.

Acknowledgements

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