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MONOPOLISTIC COMPETITION MODEL WITH ENTRANCE FEE

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Abstract: We study the monopolistic competition model with producer-retailer-consumers two-level interaction. The industry is organized according to the Dixit–Stiglitz model. The retailer is the only monopolist. A quadratic utility function represents consumer preferences. We consider the case of the retailer's leadership; namely, we study two types of behavior: with and without the free entry condition. Earlier, we obtained the result: to increase social welfare and/or consumer surplus, the government needs to subsidize (not tax!) retailers. In the presented paper, we develop these results for the situation when the producer imposes an entrance fee for retailers.

Keywords: Monopolistic competition, Retailing, Equilibrium, Taxation, Entrance fee, Social welfare, Consumer surplus.

1. Introduction

The modeling the relations of market agents (producer, retailer, consumer, etc.) can be carried out in various ways. Let us distinguish the main clusters.

Firstly, the models may be characterized by different ways of interaction between participants in competitive relationships. In particular, the "leader-follower" model was firstly studied in detail in the classical works of Stackelberg. The model considers the case of leadership in terms of output under the conditions of an asymmetric duopoly, where one of the firms makes its choice before the other.

Secondary, the models of market spatial differentiation. In the spatial models theme, the foundational work is Hotelling's linear city model [9]. In this case, the horizontal differentiation of goods is characterized by the geographical location of the producer on a unit interval. Besides, the transport costs for the delivery of goods to the consumer are imposed. While consumers are distributed over the interval evenly, their preferences are asymmetric. Note that the number of firms in the market assumes constant. Another example of this class of models is the Salop model [12] (circular city model) with one producer and several retailers located along a circle (street) at an equal distance between each other; consumers are evenly distributed along the circle and have the same preferences.

Thirdly, the models are characterized by different types of utility functions. The work of Perry and Groff [11] considers the CES utility function and evaluates the impact of integration on the change in the level of social welfare. In the work of Ottaviano, Tabuchi, and Thisse [10], a model of economic geography with a quadratic utility function is studied: here several production factors, as well as transport costs, are considered.

Fourthly, markets with different types of product differentiation, horizontal or vertical, are considered in the work of Gabzewicz and Thisse [8]. For the different locations of the stores (the firms), the authors study the conditions for the equilibrium existence.

In the presented paper, the industry is organized according to the Dixit–Stiglitz model [6, 7] with a quadratic utility function (cf. [10]). The quadratic utility function generates the linear demand function. The model is supplemented by a monopoly retailer. This way, we model a two-level interaction. The mass (the number) of producers is quite large (cf. [4, 5]). We study two types of retailer behavior: with and without the condition of free entry (cf. [1–3]). The result is that when the retailer imposes an entrance fee for each producer, it leads to an increase in both social welfare and consumer surplus.

Since the proofs of many statements are voluminous and rather technical, we give only proofs of some propositions. As to the other propositions, we provide only the schemes or the main ideas of proofs.

This paper continues the works [13, 14]. More precisely, in [14], we compared different types of interaction; as a result, we considered the situation from the point of view of the manufacturer, retailer, consumers, and society as a whole. In [13], we considered the case of retailer leadership; we studied two situations of retailer behavior: with free entry conditions and without free entry conditions; it turned out that social welfare increases when the retailer is stimulated by subsidies; a similar situation arises when considering consumer surplus.

In the present paper, we supplement the results of [13, 14] by introducing an entrance fee for the producer.

2. Model

We study the producer-retailer-consumers two-level interaction, monopolistic competition model. The model adopts several assumptions (see [13]) of monopolistic competition.

It is assumed that the number (mass) of firms is sufficiently large. Each firm produces only one type of product and sets its price. Firms produce goods of the same type ("variety") that are not absolutely substitutable. For firms, the free entry (zero profit) condition is assumed.

Goods on the market are represented by horizontally differentiated products. There are also other products on the market designated as "numéraire." In addition, it is assumed that there are several identical consumers, and each consumer supplies one unit of labor to the market.

Producers sell products through a monopoly retailer, which increases the retail price of goods by adding a mark-up.

We consider the case of linear demand corresponding to the quadratic utility function proposed by Ottaviano, Tabuchi, and Thisse [10]:

$$U(\mathbf{q}, N, A) = \alpha \int_0^N q(i)di - \frac{\beta - \gamma}{2} \int_0^N (q(i))^2\, di - \frac{\gamma}{2} \left(\int_0^N q(i)di \right)^2 + A, \tag{2.1}$$

where $\alpha > 0$, $\beta > \gamma > 0$ are some parameters[1]; N is the length of the product line, reflecting the range (interval) of varieties; $q(i) \geq 0$ is the consumption volume of variety i, $i \in [0, N]$; and $A \geq 0$ is the consumption of other aggregated products ("numéraire").

We introduce the notations: $\mathbf{q} = (q(i))_{i\in[0,N]}$ is an infinite-dimensional vector (profile) of the volume of goods; $\mathbf{p} = (p(i))_{i\in[0,N]}$ is the price profile; and $\mathbf{r} = (r(i))_{i\in[0,N]}$ is the trade mark-up profile.

Let us formulate the budget constraint

$$\int_0^N (p(i) + r(i))q(i)di + P_A A \leq wL + \int_0^N \pi_{\mathcal{M}}(i)di + \pi_{\mathcal{R}}, \tag{2.2}$$

[1]The economic meaning of the parameters can be found in the source [10, p. 413].

the right-hand side of (2.2) represents the gross domestic product (GDP) by income, and the left-hand side is expenditure. Here $p(i)$ is the wholesale price of variety i; $r(i)$ is the retailer's mark-up for product variety i; $p(i)+r(i)$ is the price of variety i for the consumer, $w \equiv 1$ is the wage rate in the industry, normalized to one; P_A is the price of other goods ("numéraire"), $\pi_{\mathcal{M}}(i)$ is the profit of the firm $i \in [0,N]$, while $\pi_{\mathcal{R}}$ is the retailer's profit.

Let us formulate the representative consumer problem:

$$\begin{cases} U(\mathbf{q},N,A) \to \max\limits_{\mathbf{q},A}, \\ \displaystyle\int_0^N p_{\mathcal{R}}(i)q(i)di + A \le L + \int_0^N \pi_{\mathcal{M}}(i)di + \pi_{\mathcal{R}}, \end{cases} \tag{2.3}$$

where $U(\mathbf{q},N,A)$ is defined in (2.1), and the price of other products P_A and wage rate w in (2.2) are normalized to one.

The problem (2.3) is solved using the Lagrange function. As a result, consumption characteristics can be determined for each $i \in [0,N]$:

$$q(i) = a - (b + cN)(p(i) + r(i)) + cP, \tag{2.4}$$

where the coefficients a, b, and c are defined as

$$a = \frac{\alpha}{\beta + (N-1)\gamma}, \quad b = \frac{1}{\beta + (N-1)\gamma}, \quad c = \frac{\gamma}{(\beta-\gamma)(\beta + (N-1)\gamma)},$$

and P is the price index

$$P = \int_0^N (p(j) + r(j))dj.$$

Let, as in [13, 14], d be the producer's marginal costs and F be the producer's fixed costs.

Then the problem of maximizing the firm's profit $i \in [0,N]$ can be written as

$$\pi_{\mathcal{M}}(i) = (p(i) - d)q(i, \mathbf{p} + \mathbf{r}) - F \ \to \ \max_{\mathbf{p}}, \tag{2.5}$$

where $q(i)$ is defined in (2.4).

Note that (2.5) is quadratic in $p(i)$.

Now let us formulate the retailer problem. Similarly to the producer problem (2.5) (see [13, 14]), let $d_{\mathcal{R}}$ be the retailer's marginal costs and $F_{\mathcal{R}}$ be the retailer's fixed costs. Let $p^*(i, r(i), N, P)$ be the optimal pricing policies, then the demand is $q(i, r(i), N, P)$ while the profile of mark-up is $\mathbf{r} = (r(i))_{i \in [0,N]}$. Then the problem of maximizing the retailer's profit is

$$\begin{cases} \pi_{\mathcal{R}} = \displaystyle\int_0^N (r(j) - d_{\mathcal{R}})\, q(j)dj - \int_0^N F_{\mathcal{R}} dj \ \to \ \max_{\mathbf{r},N}, \\ \pi_{\mathcal{M}}(p^*(i, r(i), N, P), r(i), N) \ge 0, \quad i \in [0,N]. \end{cases} \tag{2.6}$$

Due to the assumption that the firms are identical, two cases are possible when solving the problem (2.6), namely

- the free entry condition is not taken into account, i.e., $\pi_{\mathcal{M}}(i) > 0$;
- the free entry condition is taken into account, i.e., $\pi_{\mathcal{M}}(i) = 0$.

The Stackelberg equilibrium under the retailer's leadership is considered.

Let us denote the case of the retailer's leadership with the free entry condition as RL, and the case of the retailer's leadership without taking into account the free entry condition as $RL(I)$. These cases are described in detail in [13, 14].

Case RL. *The retailer simultaneously chooses a trade mark-up* $\mathbf{r} = (r(i))_{i\in[0,N]}$ *and a mass of firms* N*, correctly predicting the subsequent response of the producers.*

Case RL(I). *The retailer first uses the free entry condition to calculate* $N = N(\mathbf{r})$*, taking into account the subsequent response of producers, and then maximizes its profit through a trade mark-up* $\mathbf{r}$.

It turned out that which particular case (RL or RL(I)) arises is completely determined by the parameter

$$\mathcal{F} = \frac{F_{\mathcal{R}}}{2F}. \tag{2.7}$$

This allows us to formulate the following proposition.

Proposition 1. *1. The case RL is possible if and only if* $\mathcal{F} > 1$.

2. The case RL(I) is possible if and only if $\mathcal{F} \leq 1$.

The next proposition describes the Stackelberg equilibrium in the case of the retailer's leadership. Let

$$\Delta = \sqrt{\frac{F}{\beta-\gamma}} > 0, \quad \varepsilon = \frac{\beta-\gamma}{\gamma} > 0, \tag{2.8}$$

$$f = \sqrt{F\cdot(\beta-\gamma)} > 0, \quad D = \frac{\alpha - d - d_{\mathcal{R}}}{\sqrt{F\cdot(\beta-\gamma)}}. \tag{2.9}$$

Proposition 2. *In the cases RL and RL(I), the equilibrium demand* q*, wholesale price* p*, trade mark-up* r*, mass of firms* N*, and the retailer's profit* $\pi_{\mathcal{R}}$ *are presented in Tables 1 and 2, where* $\mathcal{F}, \Delta, \varepsilon, f,$ *and* D *are defined in (2.7)–(2.9).*

Table 1. Equilibrium in different cases of the retailer's leadership

	q	p	r	N
RL	$\Delta\sqrt{\mathcal{F}}$	$d + f\sqrt{\mathcal{F}}$	$d_{\mathcal{R}} + f\cdot\dfrac{D}{2}$	$\dfrac{\varepsilon}{2}\cdot\left(\dfrac{D}{\sqrt{\mathcal{F}}} - 4\right)$
RL(I)	Δ	$d+f$	$d_{\mathcal{R}} + f\cdot\left(\dfrac{D}{2} + \mathcal{F} - 1\right)$	$\dfrac{\varepsilon}{2}\cdot(D - 2\mathcal{F} - 2)$

Table 2. The retailer's profit in different cases of the retailer's leadership

	$\pi_{\mathcal{R}}$
RL	$\left(D - 4\sqrt{\mathcal{F}}\right)^2\cdot\dfrac{H}{2}$
RL(I)	$(D - 2\mathcal{F} - 2)^2\cdot\dfrac{H}{2}$

2.1. Entrance fee

The relationship between producers and retailers is actually regulated. As a rule, the producer must pay the retailer. Let us denote the entrance fee by F_{EF}. Then the fixed costs of the producer and the retailer will change as follows:

$$\breve{F} = F + F_{EF},$$
$$\breve{F_{\mathcal{R}}} = F_{\mathcal{R}} - F_{EF}.$$

Taking into account F_{EF}, we write the profit of the ith producer as

$$\pi_{\mathcal{M}}(i) = (p(i) - d)q(i) - (F + F_{EF})$$

and the retailer's profit as

$$\pi_{\mathcal{R}} = \int_0^N (r(i) - d_{\mathcal{R}})\, q(i)di - \int_0^N (F_{\mathcal{R}} - F_{EF})di.$$

We get the following retailer's profit optimization problem:

$$\begin{cases} \pi_{\mathcal{R}} = \displaystyle\int_0^N (r(i) - d_{\mathcal{R}})\, q(i)di - \int_0^N (F_{\mathcal{R}} - F_{EF})di \;\to\; \max_{\mathbf{r},N,F_{EF}}, \\ \pi_{\mathcal{M}}(i) = (p(i) - d)q(i) - (F + F_{EF}) \geq 0. \end{cases} \tag{2.10}$$

In what follows, we will need the following lemmas.

Lemma 1. *The optimal trade mark-up r is the same for all producers and is expressed in terms of N as follows:*

$$r = r(N) = \frac{N(\alpha - d - d_{\mathcal{R}})}{2(N + \varepsilon)} + d_{\mathcal{R}}.$$

To prove Lemma 1, it is necessary to solve the optimization problem (2.10). Due to the fact that

$$\frac{\partial \pi_{\mathcal{R}}}{\partial F_{EF}} = N > 0,$$

F_{EF} the optimum of the objective function is reached at the boundary, that is, $\pi_{\mathcal{M}}(i) = 0$. Substituting F_{EF} into $\pi_{\mathcal{R}}$, we find the maximum of the function $\pi_{\mathcal{R}}$ over the variables r and N. We solve the optimization problem by the method of needle variations; as a result, we determine the optimal mark-up of the retailer.

Lemma 2. *Under the symmetric equilibrium, the wholesale price p and the demand q are as follows:*

$$p = p(N) = \frac{\varepsilon(\alpha - d - d_{\mathcal{R}})}{2(N + \varepsilon)} + d,$$

$$q = q(N) = \frac{(\alpha - d - d_{\mathcal{R}})}{2\gamma(N + \varepsilon)} + d.$$

P r o o f. Under the symmetric equilibrium, solving the problem of the producer's profit maximization, we have

$$q(r, N) = \frac{(b + cN)(a - b(r + d))}{2b + cN}, \tag{2.11}$$

$$p(r,N) = \frac{q(r,N)}{b+cN} + d. \tag{2.12}$$

Note that

$$\frac{a}{b} = \alpha, \quad \frac{b}{c} = \frac{\beta-\gamma}{\gamma} = \varepsilon, \quad b + cN = \frac{1}{\beta-\gamma}.$$

Hence, due to Lemma 1, (2.11) is

$$\begin{aligned} q(N) = \frac{\alpha - r - d}{\gamma(N+2\varepsilon)} &= \frac{\alpha - \dfrac{N(\alpha - d - d_{\mathcal{R}})}{2(N+\varepsilon)} - d_{\mathcal{R}} - d}{\gamma(N+2\varepsilon)} = \\ &= \frac{(\alpha - d - d_{\mathcal{R}})(2N+2\varepsilon - N)}{2\gamma(N+\varepsilon)(N+2\varepsilon)} = \frac{\alpha - d - d_{\mathcal{R}}}{2\gamma(N+\varepsilon)}. \end{aligned} \tag{2.13}$$

Substituting (2.13) into (2.12), we get

$$p(N) = \frac{(\alpha - d - d_{\mathcal{R}})\varepsilon}{2(N+\varepsilon)} + d.$$

□

Proposition 3. *When an entrance fee is introduced, the equilibrium demand q, wholesale price p, trade mark-up r, entrance fee F_{EF}, mass of firms N, and retailer profit $\pi_{\mathcal{R}}$ are presented in Tables 3 and 4, where $\mathcal{F}, \Delta, \varepsilon, f$, and D are defined in* (2.7)–(2.9).

Table 3. Equilibrium in the case of introduction of an entrance fee

	q	p	r
EF	$\Delta\sqrt{2\mathcal{F}+1}$	$f\sqrt{2\mathcal{F}+1}+d$	$d_{\mathcal{R}} + f \cdot \dfrac{D}{2} - f\sqrt{2\mathcal{F}+1}$

Table 4. The retailer's fixed costs, mass of firms, and the retailer's profit with an entrance fee

	F_{EF}	N	$\pi_{\mathcal{R}}$
EF	$F_{\mathcal{R}}$	$\dfrac{\varepsilon}{2} \cdot \left(\dfrac{D}{\sqrt{2\mathcal{F}+1}} - 2\right)$	$(D - 2\sqrt{2\mathcal{F}+1})^2 \cdot \dfrac{H}{2}$

P r o o f. From Lemmas 1 and 2, we get the following simplified retailer's profit optimization problem:

$$\begin{cases} \pi_{\mathcal{R}} = N\left((r(N) - d_{\mathcal{R}})q(N) - F_{\mathcal{R}} + F_{EF}\right) \ \rightarrow \ \max\limits_{N, F_{EF}}, \\ \pi_{\mathcal{M}} = (p(N) - d)\, q(N) - (F + F_{EF}) \geq 0. \end{cases}$$

Note that

$$\frac{\partial \pi_{\mathcal{R}}}{\partial F_{EF}} = N > 0.$$

Therefore, the optimum of the objective function is attained at the boundary, i.e., for $\pi_{\mathcal{M}} = 0$ (the entrance fee condition is fulfilled), whence we find

$$F_{EF} = F_{EF}(N) = -F + (p(N) - d)q(N).$$

Substituting $F_{EF} = F_{EF}(N)$ into $\pi_{\mathcal{R}}$, we get

$$\pi_{\mathcal{R}} = N\left((r(N) - d_{\mathcal{R}})q(N) - F_{\mathcal{R}} - F + (p(N) - d)q(N)\right) \;\to\; \max_N .$$

Since the derivative of the retailer's profit equals zero, we determine the mass of producers:

$$\frac{\partial \pi_{\mathcal{R}}}{\partial N} = 0 \Leftrightarrow N = \frac{\varepsilon}{2}\left(\frac{D}{\sqrt{2\mathcal{F}+1}} - 2\right).$$

Then we find

$$q = \Delta\sqrt{2\mathcal{F}+1}, \quad p = f\sqrt{2\mathcal{F}+1} + d,$$
$$r = d_{\mathcal{R}} + f\cdot\frac{D}{2} - f\sqrt{2\mathcal{F}+1}, \quad F_{EF} = F_{\mathcal{R}}, \quad \pi_{\mathcal{R}} = (D - 2\sqrt{2\mathcal{F}+1})^2\cdot\frac{H}{2}.$$

□

3. Social welfare and consumer surplus

In this section, we consider the functions of social welfare and consumer surplus and calculate the equilibrium social welfare and equilibrium consumer surplus in two cases: under the retailer's leadership and with an entrance fee.

3.1. Social welfare

Consider the social welfare function W as a measure of the welfare of society. In the symmetric case, W has the form

$$W = (\alpha - d - d_{\mathcal{R}})Nq - \frac{\beta-\gamma}{2}\cdot Nq^2 - \frac{\gamma}{2}\cdot N^2q^2 - (F + F_{\mathcal{R}})N. \tag{3.1}$$

In various equilibrium cases, we can formulate the following proposition for the social welfare function W.

Proposition 4. *The equilibrium social welfare under the retailer's leadership and with an entrance fee is presented in Table 5, where*

$$H = \frac{F\cdot(\beta-\gamma)}{2\gamma} > 0, \tag{3.2}$$

while $\mathcal{F}, f$, and D are defined in (2.7) *and* (2.9).

We can prove Proposition 4 directly by substituting the equilibrium solutions from Proposition 2 and Proposition 3 into (3.1). After appropriate calculations, we get the results presented in Table 5.

Table 5. Social welfare in different equilibrium cases

	W
RL	$\left(D-4\sqrt{\mathcal{F}}\right)\cdot\left(\frac{3}{4}\cdot(D-2\sqrt{\mathcal{F}})-\frac{1}{\sqrt{\mathcal{F}}}\right)\cdot H$
RL(I)	$(D-2\mathcal{F}-2)\cdot\left(\frac{3}{4}\cdot(D-2\mathcal{F})-1\right)\cdot H$
EF	$\left(D-2\sqrt{2\mathcal{F}+1}\right)\cdot\left(3D-4\sqrt{2\mathcal{F}+1}\right)\cdot\frac{H}{4}$

3.2. Consumer surplus

The consumer surplus CS is a measure of the well-being that consumers derive from the consumption of goods and services. In the case of symmetric equilibrium, it is represented in the form

$$CS=\alpha Nq-\frac{\beta-\gamma}{2}Nq^2-\frac{\gamma}{2}N^2q^2-(p+r)Nq. \tag{3.3}$$

For the consumer surplus function, for the various equilibrium cases, the following proposition can be formulated.

Proposition 5. *The equilibrium consumer surplus under the retailer's leadership and with an entrance fee is presented in Table 6, where $\mathcal{F}, f, D$, and H are defined in (2.7), (2.9), and (3.2).*

Table 6. Consumer surplus in different equilibrium cases

	CS
RL	$\left(D-4\sqrt{\mathcal{F}}\right)\cdot\left(D-2\sqrt{\mathcal{F}}\right)\cdot\frac{H}{4}$
RL(I)	$(D-2\mathcal{F}-2)\cdot(D-2\mathcal{F})\cdot\frac{H}{4}$
EF	$D\cdot\left(D-2\sqrt{2\mathcal{F}+1}\right)\cdot\frac{H}{4}$

We can prove Proposition 5 directly by substituting the equilibrium solutions from Proposition 2 and Proposition 3 into (3.3). After appropriate calculations, we get the results presented in Table 6.

4. Comparison of RL and EF cases

In this section, we compare the obtained values $p, q, r, \pi_{\mathcal{R}}, W$, and CS in the case of the retailer's leadership and in the case of an entrance fee (see Tables 1–6).

We get the following result, where the indices "RL" and "EF" mean that the corresponding values are calculated for the case of the retailer's leadership and for the case of an entrance fee, respectively.

Proposition 6. *For the equilibrium price p, mark-up r, retail price $p+r$, individual consumption q, total consumption Q, welfare W, consumer surplus CS, and the retailer's profit $\pi_{\mathcal{R}}$, we get*

- $p^{EF} > p^{RL}$, *i.e., the introduction of an entrance fee always increases the wholesale price, thereby offsetting the costs of the producer*;
- $r^{EF} < r^{RL}$, *i.e., the introduction of an entrance fee reduces the trade mark-up*;
- $p^{EF} + r^{EF} < p^{RL} + r^{RL}$, *i.e., the introduction of an entrance fee an entails a decrease in the retail price*;
- $q^{EF} > q^{RL}$, *i.e., the introduction of an entrance fee entails an increase in the individual consumption*;
- $Q^{EF} > Q^{RL}$, *where* $Q = qN$, *i.e., the introduction of an entrance fee increases the total consumption*;
- $W^{EF} > W^{RL}$, $CS^{EF} > CS^{RL}$, $\pi_{\mathcal{R}}^{EF} > \pi_{\mathcal{R}}^{RL}$, *i.e., the introduction of an entrance fee leads to an increase in the social welfare, consumer surplus, and the retailer's profit.*

P r o o f. Let us prove that $CS^{EF} \geq CS^{RL}$ (the rest can be proven in a similar way). In the case when $\mathcal{F} > 1$, we have

$$\left\{\begin{array}{lcl} N^{EF} & \geq & 0, \\ N^{RL} & \geq & 0, \end{array}\right. \quad \Leftrightarrow \quad \left\{\begin{array}{lcl} D & \geq & 4\sqrt{\mathcal{F}}, \\ D & \geq & 2\sqrt{2\mathcal{F}+1}. \end{array}\right.$$

Note that $2\sqrt{\mathcal{F}} \geq \sqrt{2\mathcal{F}+1}$. Hence $D \geq 4\sqrt{\mathcal{F}}$. Then

$$\begin{aligned} CS^{EF} - CS^{RL} &= \frac{H}{4} \cdot \Big(D(D - 2\sqrt{2\mathcal{F}+1}) - (D - 4\sqrt{\mathcal{F}})(D - 2\sqrt{\mathcal{F}}) \Big) = \\ &= \frac{H}{4} \cdot \Big(D^2 - 2D\sqrt{2\mathcal{F}+1} - D^2 + 6D\sqrt{\mathcal{F}} - 8\mathcal{F} \Big) = \\ &= \frac{H}{4} \cdot \left(D(\underbrace{6\sqrt{\mathcal{F}} - 2\sqrt{2\mathcal{F}+1}}_{\geq 0}) - 8\mathcal{F} \right) \geq \\ &\geq \frac{H}{4} \cdot \Big(4\sqrt{\mathcal{F}}(6\sqrt{\mathcal{F}} - 2\sqrt{2\mathcal{F}+1}) - 8\mathcal{F} \Big) = \\ &= 2H \cdot \Big(\sqrt{\mathcal{F}}(3\sqrt{\mathcal{F}} - \sqrt{2\mathcal{F}+1}) - \mathcal{F} \Big) = \\ &= 2H \cdot \sqrt{\mathcal{F}}\Big(2\sqrt{\mathcal{F}} - \sqrt{2\mathcal{F}+1} \Big) \geq 0, \end{aligned}$$

i.e., we get $CS^{EF} > CS^{RL}$ for $\mathcal{F} > 1$.

In the case when $0 < \mathcal{F} \leq 1$, we have

$$\left\{\begin{array}{lcl} N^{EF} & \geq & 0, \\ N^{RL} & \geq & 0, \end{array}\right. \quad \Leftrightarrow \quad \left\{\begin{array}{lcl} D & \geq & 2(\mathcal{F}+1), \\ D & \geq & 2\sqrt{2\mathcal{F}+1}. \end{array}\right.$$

Note that $\mathcal{F} + 1 \geq \sqrt{2\mathcal{F}+1}$. Hence $D \geq 2(\mathcal{F}+1)$. Then

$$\begin{aligned} CS^{EF} - CS^{RL} &= \frac{H}{4} \cdot \Big(D(D - 2\sqrt{2\mathcal{F}+1}) - (D - 2\mathcal{F} - 2)(D - 2\mathcal{F}) \Big) = \\ &= \frac{H}{4} \cdot \Big(D^2 - 2D\sqrt{2\mathcal{F}+1} - D^2 + 2D\mathcal{F} + (D - 2\mathcal{F})(2\mathcal{F} + 2) \Big) = \end{aligned}$$

$$
\begin{aligned}
&= \frac{H}{4} \cdot \Big(D(\underbrace{4\mathcal{F}+2-2\sqrt{2\mathcal{F}+1}}_{\geq 0}) - 2\mathcal{F}(2\mathcal{F}+2)\Big) \geq \\
&\geq \frac{H}{4} \cdot \Big(2(\mathcal{F}+1)(4\mathcal{F}+2-2\sqrt{2\mathcal{F}+1}) - 2\mathcal{F}(2\mathcal{F}+2)\Big) = \\
&= \frac{H}{4} \cdot 4\Big((\mathcal{F}+1)(2\mathcal{F}+1-\sqrt{2\mathcal{F}+1}) - \mathcal{F}(\mathcal{F}+1)\Big) = r \\
&= H \cdot (\mathcal{F}+1)\left(\mathcal{F}+1-\sqrt{2\mathcal{F}+1}\right) \geq 0,
\end{aligned}
$$

i.e., $CS^{EF} > CS^{RL}$ for $0 < \mathcal{F} \leq 1$. □

As for the values N^{EF} and N^{RL}, there are two possibilities depending on the values $\mathcal{F}$ and D, see Fig. 1. In Fig. 1, only the areas $N^{EF} < N^{RL}$ and $N^{EF} > N^{RL}$ are of interest, since the number

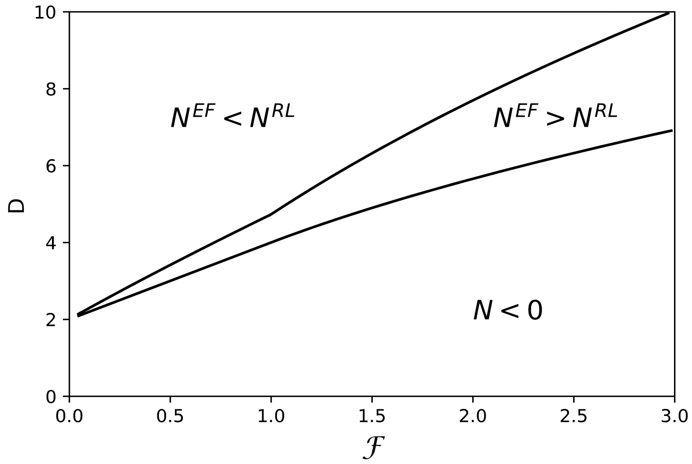

Figure 1. Comparison of values N for RL and EF cases.

(mass) of producers in these areas is non-negative. An analytical representation of these areas is

$$
\begin{aligned}
&N^{EF} < N^{RL}, \quad D > \underline{D}, \\
&N^{EF} > N^{RL}, \quad \overline{D} > D > \underline{D},
\end{aligned}
$$

where

$$
\underline{D} = \begin{cases} 4\sqrt{\mathcal{F}} & \text{if } \mathcal{F} > 1; \\ 2(\mathcal{F}+1) & \text{if } \mathcal{F} \leq 1, \end{cases}
$$

$$
\overline{D} = \begin{cases} \left(1+\sqrt{\dfrac{1}{\mathcal{F}}+2}\right) \cdot \sqrt{1+2\mathcal{F}} \cdot \dfrac{2\mathcal{F}}{1+\mathcal{F}} & \text{if } \mathcal{F} > 1; \\ \left(1+\sqrt{1+2\mathcal{F}}\right) \cdot \sqrt{1+2\mathcal{F}} & \text{if } \mathcal{F} \leq 1. \end{cases}
$$

5. Conclusion

The presented paper analyzes the monopolistic competition trade model with two-level interaction. The situation of the retailer's leadership is considered in detail. We show that, under the retailer's leadership, two ways are possible depending on $\mathcal{F}$: artificially restricted and unrestricted market. The parameter $\mathcal{F}$ is the ratio of the retailer's fixed costs to the twice fixed costs of each producer. In the case of an artificially limited market, the retailer independently restricts the entry of producers. Otherwise (i.e., in the case of an unrestricted market), the free entry condition is used, which means that producers enter the market until their profits become zero.

In addition, we study the possible effects when the retailer imposes the market entrance fee for producers. We show that the introduction of an entrance fee by the retailer is justifiable since it increases the social welfare and consumer surplus, as well as the retailer's profit.

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