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NOTE ON SUPER $(a,1)$–P_3–ANTIMAGIC TOTAL LABELING OF STAR S_n

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Abstract: Let $G=(V,E)$ be a simple graph and H be a subgraph of G. Then G admits an H-covering, if every edge in $E(G)$ belongs to at least one subgraph of G that is isomorphic to H. An $(a,d)-H$-antimagic total labeling of G is bijection $f:V(G)\cup E(G)\rightarrow\{1,2,3,\ldots,|V(G)|+|E(G)|\}$ such that for all subgraphs H' of G isomorphic to H, the H' weights $w(H')=\sum_{v\in V(H')}f(v)+\sum_{e\in E(H')}f(e)$ constitute an arithmetic progression $\{a,a+d,a+2d,\ldots,a+(n-1)d\}$, where a and d are positive integers and n is the number of subgraphs of G isomorphic to H. The labeling f is called a super $(a,d)-H$-antimagic total labeling if $f(V(G))=\{1,2,3,\ldots,|V(G)|\}$. In [5], David Laurence and Kathiresan posed a problem that characterizes the super $(a,1)-P_3$-antimagic total labeling of Star S_n, where $n=6,7,8,9$. In this paper, we completely solved this problem.

Keywords: H-covering, Super $(a,d)-H$-antimagic, Star.

1. Introduction

Let $G=(V(G),E(G))$ and $H=(V(H),E(H))$ be simple and finite graphs. Let $|V(G)|=v_G$, $|E(G)|=e_G$, $|V(H)|=v_H$ and $|E(H)|=e_H$. An edge covering of G is a family of different subgraphs $H_1,H_2,H_3,\ldots,H_k$ such that any edge of $E(G)$ belongs to at least one of the subgraphs H_j, $1\leq j\leq k$. If the $H_j^{'}$s are isomorphic to a given graph H, then G admits an H-covering. Gutienrez and Lladó [2] defined $H-$magic labeling, which is a generalization of Kotzig and Rosa's edge magic total labeling [4]. A bijection $f:V(G)\cup E(G)\rightarrow\{1,2,3,...,v_G+e_G\}$ is called an H-magic labeling of G if there exists a positive integer k such that each subgraph H' of G isomorphic to H satisfies

$$w(H')=\sum_{v\in V(H')}f(v)+\sum_{e\in E(H')}f(e)=k.$$

In this case, they say that G is H-magic. When $f(V(G))=\{1,2,3,\ldots,v_G\}$, we say that G is $H-$super magic. On the other hand, Inayah et al. [3] introduced $(a,d)-H$-antimagic total labeling of G which is defined as a bijection $f:V(G)\cup E(G)\rightarrow\{1,2,3,\ldots,v_G+e_G\}$ such that for all subgraphs H' of G isomorphic to H, the set of H'-weights

$$w(H')=\sum_{v\in V(H')}f(v)+\sum_{e\in E(H')}f(e)$$

constitutes an arithmetic progression $a,a+d,a+2d,\ldots,a+(n-1)d$, where a and d are some positive integers and n is the number of subgraphs isomorphic to H. In this case, they say that G is $(a,d)-H-$antimagic. If $f(V(G))=\{1,2,3,\ldots,v_G\}$, they say that f is a super $(a,d)-H$-antimagic total labeling and G is super $(a,d)-H$-antimagic. This labeling is a more general case of super (a,d)-edge-antimagic total labelings. If $H\cong K_2$, then we say that super $(a,d)-H$-antimagic labelings, which is also called super (a, d)-edge-antimagic total labelings and have been introduced in [6]. They studied some basic properties of such labeling and also proved the following theorem.

Theorem 1 [3]. *If G has a super $(a, d) - H$-antimagic total labeling and t is the number of subgraphs of G isomorphic to H, then G has a super $(a', d) - H$-antimagic total labeling, where $a' = [(v_G + 1)v_H + (2v_G + e_G + 1)e_H] - a - (t - 1)d$.*

Several authors are studied antimagic type labeling of graphs see [1]. In 2015, Laurence and Kathiresan [5] obtained an upper bound of d for any graph G, and they investigated the existence of super $(a, d) - P_3$-antimagic total labeling of star graph S_n. First, they observed that S_n admits a P_h-covering for $h = 2, 3$, and the star S_n contains

$$t = \binom{n}{h-1}$$

subgraphs P_h, $h = 2, 3$, which is denoted by P_h^j, $1 \leq j \leq h$. In 2005, Sugeng et al. [7] investigated the case $h = 2$ using super (a, d)-edge-antimagic total labeling. In 2015, the case of $h = 3$ was investigated by Laurence and Kathiresan [5]. Here they observed that if the star $S_n, n \geq 3$ admits a super $(a, d) - P_3$-antimagic total labeling then $d \in \{0, 1, 2\}$. Now, they proved the star S_n, $n \geq 3$ has super $(4n + 7, 0) - P_3$-antimagic total labeling and S_n, $n \geq 3$ admits a super $(a, 2) - P_3$-antimagic total labeling if and only if $n = 3$. Also, they proved the following theorems and posed a problem.

Theorem 2 [5]. *If the star S_n, $n \geq 3$ has super $(a, 1)$-P_3-antimagic total labeling, then $3 \leq n \leq 9$. Moreover, the star S_n admits a super $(a, 1)$-P_3-antimagic total labeling, where $a = 19$, for $n = 3$ and $a = 21$, for $n = 4$.*

Theorem 3 [5]. *For $n = 5$, the star S_n has no super $(a, 1)$-P_3-antimagic total labeling.*

Problem 1. [5] *For each $n, 6 \leq n \leq 9$ characterize the super $(a, 1) - P_3$-antimagic total labeling for the star S_n.*

In this paper, we present the complete solution to the above problem.

2. Main Results

Let $S_n \cong K_{1,n}, n \geq 1$ be the star graph and let v_0 be the central vertex and let $v_i, 1 \leq i \leq n$ be its adjacent vertices. Thus S_n has $n + 1$ vertices and n edges.

Theorem 4. *The star S_6 has no super $(a, 1) - P_3$-antimagic total labeling.*

P r o o f. Let $V(S_6) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E(S_6) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6\}$ be the vertex set and the edge set of Star S_6. Suppose there exists a super $(a, 1) - P_3$-antimagic total labeling $f : V \cup E \to \{1, 2, 3, \ldots, 13\}$ for S_6 and let v_0 be the central vertex of S_6. In the computation of P_3 — weights the label of the central vertex v_0,$f(v_0)$ is used 15 times and label of other vertices and edges say i are used 5 times each. Therefore,

$$10f(v_0) + 5\sum_{i=1}^{13}(i) = \frac{15}{2}[2a + 14],$$

which implies $a = (70 + 2f(v_0))/3$. Since $1 \leq f(v_0) \leq 7$, it follows that $a = 24$ if $f(v_0) = 1$, $a = 26$ if $f(v_0) = 4$ and $a = 28$ if $f(v_0) = 7$.

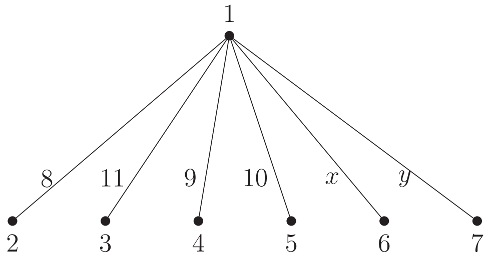

Figure 1. There is no possible to obtain P_3-weight 27.

Case (i): $f(v_0) = 1$. Then $a = 24$ and the P_3 — weights of S_6 are given by $W = \{24, 25, \ldots, 38\}$. Now, the P_3 — weight 24 is getting exactly two possible 5 elements sum $(1, 2, 4, 8, 9)$ or $(1, 2, 3, 8, 10)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_3$ or v_0v_2 is $f(e_1) = 8$ and $f(e_2) = 9$ or 10.

Subcase (i): $f(e_2 = v_0v_3) = 9$. Then $a = 24$ and hence the label of the vertices and edges are $f(v_0) = 1, f(v_1) = 2, f(v_3) = 4, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 9$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 3, 8, 11)$ and hence the label of an edge $e_3 = v_0v_2$ is $f(e_3) = 11$. Also,the P_3 — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 10)$ and hence the label of an edge $e_4 = v_0v_4$ is $f(e_4) = 10$.

Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of S_6 (see Fig. 1). Clearly, the label of the edges x and y is $f(x) = 12$ or 13 and $f(y) = 13$ or 12. If $f(x) = 12$ then $f(y) = 13$ and hence there is no P_3 — weight 27. Also, if $f(x) = 13$ then $f(y) = 12$ and hence there is no P_3 — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1 = 9)$ and $f(e_2) = 8$ for the P_3 — weight 24 is used to getting the P_3 — weight 27.

Subcase (ii): $f(e_2 = v_0v_2) = 10$. Then $a = 24$ and hence the label of the vertices and edges of P_3 — weight 24 is $f(v_0) = 1, f(v_1) = 2, f(v_2) = 3, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 10$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 9)$ and hence the label of an edge $e_3 = v_0v_4$ is $f(e_3) = 9$. Also, the P_3 — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 4, 8, 11)$ and hence the label of an edge $e_4 = v_0v_3$ is $f(e_4) = 11$. Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of S_6 (see Fig. 2). Clearly, the label of the edges x and y is $f(x) = 12$ or 13 and $f(y) = 13$ or 12. If $f(x) = 12$ then $f(y) = 13$ and hence there is no P_3 — weight 27. Also, If $f(x) = 13$ then $f(y) = 12$ and hence there is no P_3 — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 10$ and $f(e_2) = 8$ for the P_3 — weight 24 is used to getting the P_3 – weight 27.

Case (ii): $f(v_0) = 7$. Then $a = 28$. Now, if f is a super $(28, 1) - P_3$-antimagic total labeling of S_6, then by Theorem 1 [3], $\bar{f}$ is a super $(24, 1) - P_3$-antimagic total labeling, which does not exist by Case (i).

Case (iii): $f(v_0) = 4$. Then $a = 26$ and hence the P_3 — weights of S_6 are given by $W = \{26, 27, \ldots, 40\}$. Now, the P_3 — weight 26 is getting exactly four possibles 5 elements sum such as $(4, 1, 2, 8, 11)$, $(4, 1, 2, 9, 10)$, $(4, 2, 3, 8, 9)$ and $(4, 1, 3, 8, 10)$ and hence the edges $e_1 = v_0v_1$ or v_0v_2 and $e_2 = v_0v_2$ or v_0v_3 with $f(e_1) = 8$ or 9 and $f(e_2) = 9$ or 10 or 11.

Subcase (i): $f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 11$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 8$ and

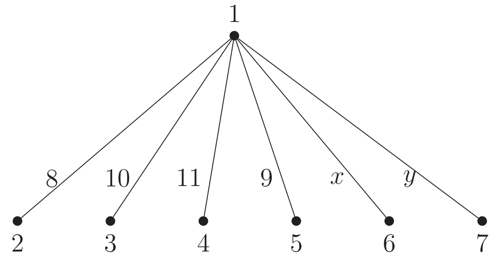

Figure 2. The possible edge labels x and y are obtain P_3-weight 27.

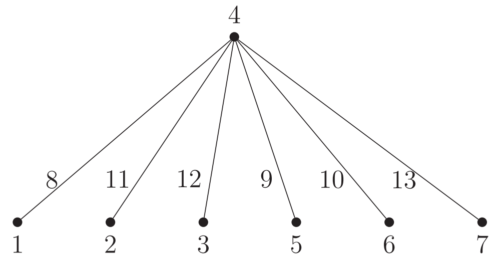

Figure 3. There is no possible to obtain P_3-weight 30.

$f(e_2 = v_0v_2) = 11$. Now, the P_3 — weight 27,28 and 29 are getting exactly one possible 5 elements sum $(4, 1, 5, 8, 9)$,$(4, 1, 3, 8, 12)$ and $(4, 1, 6, 8, 10)$. Hence the label of the edges $e_3 = v_0v_3$, $e_4 = v_0v_4$, $e_5 = v_0v_5$ and $e_6 = v_0v_6$ is $f(e_3) = 12, f(e_4) = 9, f(e_5) = 10$ and $f(e_6) = 13$. From Fig. 3, there is no P_3 — weight is 30, which is a contradiction.

A similar contradiction arises, if the edges e_1 and e_2 with $f(e_1 = v_0v_1) = 11$ and $f(e_2 = v_0v_2) = 8$ for P_3 — weight 26 are used to getting the P_3 — weight 33, for more details see Fig. 4.

Subcase (ii): $f(e_1 = v_0v_1) = 9$ and $f(e_2 = v_0v_2) = 10$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 9$ and $f(e_2 = v_0v_2) = 10$. Now, the P_3 — weight 27 is getting exactly two possibles 5 elements sum such as $(4, 2, 3, 10, 8)$, $(4, 1, 5, 9, 8)$ and hence the label of the edges $e_3 = v_0v_3$ or v_0v_4 is $f(e_3) = 8$. If an edge $e_3 = v_0v_3$ with $f(e_3) = 8$ then we get the P_3 — weight as sum of 5 elements $(4, 1, 3, 9, 8)$ is 25, which is a contradiction. If an edge $e_3 = v_0v_4$ with $f(e_3) = 8$ then we get the P_3 — weights from 28 to 32 are getting exactly one possible 5 elements sum such as $(4, 1, 3, 9, 11), (4, 2, 5, 10, 8), (4, 2, 3, 10, 11), (4, 3, 5, 11, 8)$ and $(4, 1, 6, 9, 12)$. From Fig. 5, there is no P_3 — weight 33, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1 = v_0v_1) = 10$ and $f(e_2 = v_0v_2) = 9$ for the P_3 — weight 26 is used to getting the P_3 — weight 27, which is a contradiction.

Subcase (iii): $f(e_1 = v_0v_2) = 8$ and $f(e_2 = v_0v_3) = 9$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_2) = 2, f(v_3) = 3, f(e_1 = v_0v_2) = 8$ and $f(e_2 = v_0v_3) = 9$. Now, the P_3 — weight 27 is getting exactly one possible 5 elements sum $(4, 1, 3, 9, 10)$ and hence the label of an edge $e_3 = v_0v_1$ is $f(e_3) = 10$. Thus, we get a P_3 — weight

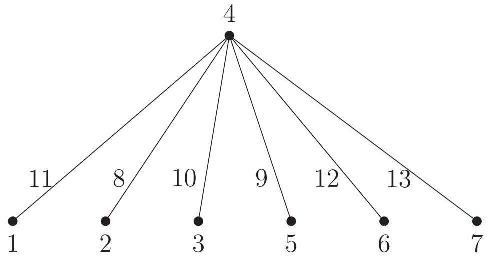

Figure 4. The possible edge label is obtain to P_3-weight 33.

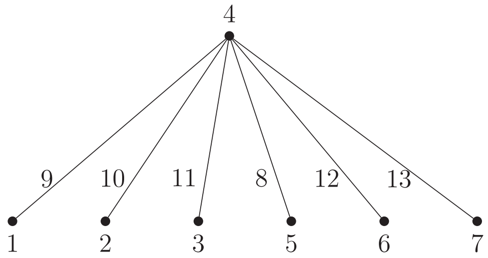

Figure 5. There is no possible to obtain P_3-weight 33.

as sum of 5 elements $(4, 1, 2, 10, 8)$ is 25, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_2$ and $e_2 = v_0v_3$ with $f(e_1 = v_0v_2) = 9$ and $f(e_2 = v_0v_3) = 8$ for the P_3 — weight 26. The P_3 — weight 27 is getting exactly one possible 5 elements sum $(4, 1, 2, 11, 9)$ and hence the label of an edge $f(e_3 = v_0v_1) = 11$. Thus, we get the $P_3 = (v_0, v_1, v_3, e_3 = v_0v_1, e_2 = v_0v_3)$ with weight $(4+1+3+11+8)$ is 27, which is a contradiction.

Subcase (iv): $f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 10$. Then $a = 26$ and hence the label of the vertices and edges of P_3 — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_3) = 3, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 10$. Now, the P_3 — weight 27 is getting exactly two possibles 5 elements sum such as $(4, 1, 2, 8, 12)$, $(4, 1, 5, 8, 9)$ and hence the label of the edges $e_3 = v_0v_2$ or v_0v_4 is $f(e_3) = 12$ or 9. If an edge $e_3 = v_0v_2$ with $f(e_3) = 12$ then the P_3 — weights 28 and 29 are getting exactly one possible 5 elements sum $(4, 1, 6, 8, 9)$ and $(4, 1, 5, 8, 11)$. From Fig. 6, there is no P_3 — weight 30, which is a contradiction. If an edge $e_4 = v_0v_4$ with $f(e_4) = 9$ then the P_3 — weight 28 is getting exactly one possible 5 elements sum $(4, 1, 2, 8, 13)$ and hence the label of an edge $e_5 = v_0v_2$ is $f(e_5) = 13$. From Fig. 7, there is no P_3 — weight 29 when $x = 11$ or 12 and $y = 12$ or 11, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_3$ with $f(e_1 = v_0v_1) = 10$ and $f(e_2 = v_0v_3) = 8$ for the P_3 — weight 26 are used to getting the P_3 — weight 27, which is a contradiction. □

Theorem 5. *The star S_7 has no super $(a, 1) - P_3$-antimagic total labeling.*

P r o o f. Let $V(S_7) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E(S_7) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6, v_0v_7\}$ be the vertex and edge set of star S_7. Suppose there exists a super $(a, 1) - P_3$-antimagic total labeling $f : V \cup E \to \{1, 2, 3, \ldots, 15\}$ for S_7 and let v_0 be the central vertex of S_7. In the

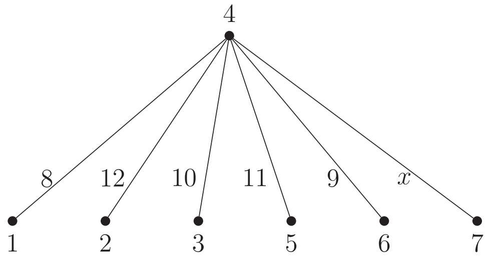

Figure 6. There is no possible to obtain P_3-weight 30.

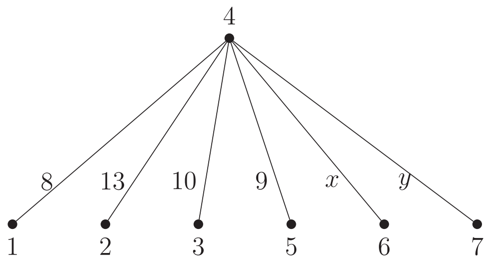

Figure 7. There is no possible to obtain P_3-weight 29.

computation of P_3 — weights the label of the central vertex v_0, $f(v_0)$ is used 21 times and label of other vertices and edges say i are used 6 times each. Therefore,

$$15f(v_0)+6\sum_{i=1}^{15}(i)=\frac{21}{2}[2a+20],$$

which implies that we get

$$a=\frac{15f(v_0)+510}{21}.$$

Since $1\le f(v_0)\le 8$, we have only two values a such as $a=25$ if $f(v_0)=1$ and $a=30$ if $f(v_0)=8$.

Case (i): $f(v_0)=1$. Then $a=25$ and the P_3 — weights of S_7 is given by $W=\{25,26,\ldots,45\}$. Now, the P_3 — weight 25 is getting exactly one possible 5 elements sum $(1,2,3,9,10)$ and hence the label of edges $e_1=v_0v_1$ and $e_2=v_0v_2$ is $f(e_1)=9$ and $f(e_2)=10$. Since the minimum possible sum of vertices labels for P_3 — weight is 7, it follows that there is no P_3 — weight 26, which is a contradiction. A similar contradiction arises, if the edges $e_1=v_0v_1$ and $e_2=v_0v_2$ with $f(e_1)=10$ and $f(e_2)=9$ for the P_3 — weight 25 is used to getting the P_3 — weight 27.

Case (ii): $f(v_0)=8$. Then $a=30$. Now, if f is a super $(30,1)-P_3$-antimagic total labeling of S_6, then by Theorem 1 [3], $\bar{f}$ is a super $(25,1)-P_3$-antimagic total labeling, which does not exist by Case (i). □

Theorem 6. *The star S_8 has no super $(a,1)-P_3$-antimagic total labeling.*

P r o o f. Let $V(S_8)=\{v_0,v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$ and $E(S_8)=\{v_0v_1,v_0v_2,v_0v_3,v_0v_4,v_0v_5,v_0v_6,v_0v_7,v_0v_8\}$ be the vertex and edge set of star S_8. Suppose there exists a super $(a,1)-P_3$-antimagic total labeling $f:V\cup E\to\{1,2,3,\ldots,17\}$ for S_8 and let v_0 be the central vertex of S_8.

In the computation of P_3 — weights the label of the central vertex v_0, $f(v_0)$ is used 28 times and label of other vertices and edges say i are used 7 times each. Therefore,

$$21f(v_0) + 7\sum_{i=1}^{17}(i) = \frac{28}{2}[2a + 27],$$

which implies that we get

$$a = \frac{21f(v_0) + 693}{28}.$$

Since $1 \le f(v_0) \le 9$, we have only two values a such as $a = 27$, if $f(v_0) = 3$ and $a = 30$, if $f(v_0) = 7$.

Case (i): $f(v_0) = 3$. Then $a = 27$ and the P_3 — weights of S_8 is given by $W = \{27, 28, \ldots, 54\}$. Now, the P_3 — weight 27 is getting exactly one possible 5 elements sum $(3, 1, 2, 10, 11)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ is $f(e_1) = 10$ and $f(e_2) = 11$. Since the minimum possible sum of vertices labels for P_3 — weight is 8, it follows that there is no P_3 — weight 29, which is a contradiction. A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 11$ and $f(e_2) = 10$ for the P_3 — weight 27 is used to getting the P_3 — weight 29.

Case (ii) $f(v_0) = 7$ Then $a = 30$. Now, if f is a super $(30, 1) - P_3$-antimagic total labeling of S_6, then by Theorem 1 [3], $\bar{f}$ is a super $(27, 1) - P_3$-antimagic total labeling, which does not exist by Case (i). □

Theorem 7. *The star S_9 has no super $(a, 1) - P_3$-antimagic total labeling.*

P r o o f. Let $V(S_9) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ be the vertex set of star S_9. Suppose there exists a super $(a, 1) - P_3$-antimagic total labeling $f : V \cup E \to \{1, 2, 3, \ldots, 19\}$ for S_9 and let v_0 be the central vertex of S_9. In the computation of P_3 — weights the label of the central vertex v_0, $f(v_0)$ is used 36 times and label of other vertices and edges say i are used 8 times each. Therefore,

$$28f(v_0) + 8\sum_{i=1}^{19}(i) = \frac{36}{2}[2a + 35],$$

which implies that we get

$$a = \frac{14f(v_0) + 445}{18}.$$

Since $1 \le f(v_0) \le 10$, we have that a is not an integer, which is a contradiction. □

From Theorem 2–3 [5], Theorem 4–7, we get the following result.

Theorem 8. *The star $S_n, n \ge 3$ admits a super $(a, 1) - P_3$-antimagic total labeling if and only if $n = 3$ and 4.*

3. Conclusion and Scope

In [5], they investigated the existence of super (a, d)-P_3-antimagic total labeling of star S_n and posed the Problem 1 [5]. This paper proved the star S_n has no super $(a, 1)$-P_3-antimagic total labeling, where $n = 6, 7, 8, 9$. Therefore, we have entirely solved Problem 1 [5].

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