ON THE COMPLETENESS PROPERTIES OF THE C-COMPACT-OPEN TOPOLOGY ON $C(X)$[1,2]

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This is a study of the completeness properties of the space $C_{rc}(X)$ of continuous real-valued functions on a Tychonoff space X, where the function space has the C-compact-open topology. Investigate the properties such as completely metrizable, Čech-complete, pseudocomplete and almost Čech-complete.

Keywords: C-compact-open topology, Set-open topology, Čech-complete, Baire space, Function space.

Introduction

The set-open topology on a family λ of nonempty subsets of the Tychonoff space X (the λ-open topology) is a generalization of the compact-open topology and of the topology of pointwise convergence. This topology was first introduced by Arens and Dugundji [7].

All sets of the form $\{f \in C(X) : \ f(F) \subseteq U\}$, where $F \in \lambda$ and U is an open subset of real line $\mathbb{R}$, form a subbase of the λ-open topology.

We denote the space $C(X)$ with λ-open topology by $C_\lambda(X)$. Note that the set-open topology and its properties depend on the family λ. So if we take a family λ of all finite, compact or pseudocompact subsets of X then we get point-open, compact-open, pseudocompact-open topology on $C(X)$ respectively. These topologies actively studied and find their application in measure theory and functional analysis.

Sure if we take an arbitrary family λ then a topological space $C_\lambda(X)$ may have weaker properties, for example, it can not be a regular or Hausdorff space.

Special interest for applications when a space $C_\lambda(X)$ is a locally convex topological vector space (TVS). Therefore, we take a "good" family λ of subsets of X which define locally convex TVS on $C(X)$. For example a families of all compact, finite, metrizable compact, sequentially compact, countable compact, pseudocompact or C-compact subsets of X are "good" families (see [18]).

Recall that a subset A of a space X is called *C-compact subset* X if, for any real-valued function f continuous on X, the set $f(A)$ is compact in $\mathbb{R}$.

Note that, in the case $A = X$, the property of the set A to be C-compact coincides with the pseudocompactness of the space X.

The space $C(X)$, equipped with the set-open topology on the family of all C-compact subsets of X, is denoted by $C_{rc}(X)$.

This article is a continuation of the article [3] on the study of topological properties of the space $C_{rc}(X)$.

The importance of studying the C-compact-open topology on $C(X)$, due to the fact that if $C_\lambda(X)$ is a locally convex TVS then the family λ consists of C-compact subsets of X.

Moreover if $C_\lambda(X)$ is a topological (even paratopological) group then the family λ consists of C-compact subsets of X.

In [19] was found to be characteristic for the space $C_\lambda(X)$ such that $C_\lambda(X)$ is a toplogical group, TVS or locally convex TVS.

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Note that if the set-open topology coincides with the topology of uniform convergence on the family λ then $C_\lambda(X)$ is a topological algebra.

Recall that the topology of uniform convergence is given by a base at each point $f \in C(X)$. This base consists of all sets $\{g \in C(X) : \sup\limits_{x\in X} |g(x)-f(x)| < \varepsilon\}$. The topology of uniform convergence on elements of a family λ (the λ-topology), where λ is a fixed family of non-empty subsets of the set X, is a natural generalization of this topology. All sets of the form $\{g \in C(X) : \sup\limits_{x\in F} |g(x) - f(x)| < \varepsilon\}$, where $F \in \lambda$ and $\varepsilon > 0$, form a base of the λ-topology at a point $f \in C(X)$. We denote the space $C(X)$ with λ-topology by $C_{\lambda,u}(X)$.

In [19] proved the following theorem (Theorem 3.3).

Theorem 0.1. *For a space X, the following statements are equivalent.*

1. $C_\lambda(X) = C_{\lambda,u}(X)$.
2. $C_\lambda(X)$ *is a topological group.*
3. $C_\lambda(X)$ *is a topological vector space.*
4. $C_\lambda(X)$ *is a locally convex topological vector space.*
5. λ *is a family of C-compact sets and $\lambda = \lambda(C)$, where $\lambda(C) = \{A \in \lambda$: for every C-compact subset B of the space X with $B \subset A$, the set $[B, U]$ is open in $C_\lambda(X)$ for any open set U of the space $\mathbb{R}\}$.*

In [3], in addition to studying some basic properties of $C_{rc}(X)$, metrizability, separability and submetrizability of $C_{rc}(X)$ have been studied. In this paper, we study various kinds of completeness of the C-compact topology such as complete metrizability, Čech-completeness, pseudocompleteness and almost Čech-completeness of $C_{rc}(X)$.

Throughout the rest of the paper, we use the following conventions. All spaces are completely regular Hausdorff, that is, Tychonoff.

The elements of the standard subbases of the λ-open topology and λ-topology will be denoted as follows:

$[F, U] = \{f \in C(X) : f(F) \subseteq U\}$,

$\langle f, F, \varepsilon\rangle = \{g \in C(X) : \sup\limits_{x\in F} |f(x) - g(x)| < \varepsilon\}$, where $F \in \lambda$, U is an open subset of $\mathbb{R}$ and $\varepsilon > 0$.

If X and Y are any two spaces with the same underlying set, then we use $X = Y$, $X \leq Y$ and $X < Y$ to indicate, respectively, that X and Y have same topology, that the topology Y is finer than or equal to the topology on X and that the topology on Y is strictly finer than the topology on X. The symbols $\mathbb{R}$ and $\mathbb{N}$ denote the spaces of real numbers and natural numbers, respectively.

We recall that a subset of X that is the complete preimage of zero for a certain function from $C(X)$ is called a zero-set. A subset O of a space X is called functionally open (or a cozero-set) if $X \setminus O$ is a zero-set.

Cover is called functionally open if it consists of functionally open subsets of X.

Let $G \subseteq C(X)$. A set $A \subseteq X$ is said to be G-bounded if $f(A)$ is a bounded subset of $\mathbb{R}$ for each $f \in G$. We say that A is bounded in X if A is G-bounded for $G = C(X)$.

A space X is called a μ-space if every closed bounded subset of X is compact. In the literature, a μ-space is also called a hyperisocompact or a Nachbin-Shirota space (NS-space for brevity).

The closure of a set A will be denoted by $\overline{A}$; the symbol $\varnothing$ stands for the empty set.

If $A \subseteq X$ and $f \in C(X)$, then we denote by $f|_A$ the restriction of the function f to the set A. As usual, $f(A)$ and $f^{-1}(A)$ are the image and the complete preimage of the set A under the mapping f, respectively.

The constant zero function defined on X is denoted by f_0. We call it the constant zero function in $C(X)$.

The remaining notation can be found in [5].

Obvious that a pseudocompact subset of X is a C-compact subset of X and a C-compact subset of X is a bounded subset of X by definition.

In [5] given a well-known Isbell-Frol$\acute{i}k$-Mr$\acute{o}$wka space in which the concepts of pseudocompactness and C-compactness differ even for closed subsets.

We note some important properties of C-compact subset (see [2] and [8]).

The subset A is an C-compact subset of X if and only if every countable functionally open (in X) cover of A has a proper subcollection whose union is dense in A.

For any Tychonoff space X, pseudocompactness equivalent to feebly compactness of X. Recall that a space X is called a feebly compact if whenever countably infinite locally finite open cover of X has a proper subcollection whose union is dense in X. It is well known that the closure of the pseudocompact (bounded) subset of X will be pseudocompact (bounded) subset of X. It holds true for C-compact set [1].

Note that for a closed subset A in a normal Hausdorff space X, the following equivalent (see [14]).

1. A is countably compact.
2. A is pseudocompact.
3. A is C-compact subset of X.
4. A is bounded.

Recall that a Tychonoff space X is called submetrizable if X admits a weaker metrizable topology.

Note that for a subset A in a submetrizable space X, the following are equivalent (see [4]).

1. A is countably compact subset of X.
2. A is pseudocompact subset of X.
3. A is sequentially compact subset of X.
4. A is C-compact subset of X.
5. A is compact subset of X.
6. A is metrizable compact subset of X.

Note that every closed bounded subset of Dieudonn$\acute{e}$ complete space is compact (see [14]).

1. Uniform Completeness of $C_{rc}(X)$

There are three ways to consider the C-compact-open topology on $C(X)$ [3].

First, one can use as subbase the family $\{[A, V] : A$ is a C-compact subset of X and V is an open subset of $\mathbb{R}\}$. But one can also consider this topology as the topology of uniform convergence on the C-compact subsets of X, in which case the basic open sets will be of the form $\langle f, F, \varepsilon\rangle$, where $f \in C(X)$, F is a C-compact subset of X and ε is a positive real number.

The third way is to look at the C-compact-open topology as a locally convex topology on $C(X)$. For each C-compact subset A of X and $\varepsilon > 0$, we define the seminorm p_A on $C(X)$ and $V_{A,\varepsilon}$ as follows: $p_A(f) = \sup\{|f(x)|\colon x \in A\}$ and $V_{A,\varepsilon} = \{f \in C(X)\colon p_A(f) < \varepsilon\}$. Let $\Psi = \{V_{A,\varepsilon}\colon A$ is a C-compact subset of X, $\varepsilon > 0\}$. Then for each $f \in C(X)$, $f + \Psi = \{f + V\colon V \in \Psi\}$ forms a neighborhood base at f. This topology is locally convex since it is generated by a collection of seminorms and it is same as the C-compact-open topology on $C(X)$. It is also easy to see that this topology is Tychonoff.

The topology of uniform convergence on the C-compact subsets of X is actually generated by the uniformity of uniform convergence on these subsets. Recall that a uniform space E is called complete provided that every Cauchy net in E converges to some element in E.

In order to characterize the uniform completeness of $C_{rc}(X)$, we need to talk about rc-continuous functions and rc_f-spaces.

D e f i n i t i o n 1.1. A function $f : X \mapsto \mathbb{R}$ is said to be rc-continuous, if for every C-compact subset $A \subseteq X$, there exists a continuous function $g : X \mapsto \mathbb{R}$ such that $g|_A = f|_A$. A space X is called a rc_f-space if every rc-continuous function on X is continuous.

Theorem 1.1. *The space $C_{rc}(X)$ is uniformly complete if and only if X is a rc_f-space.*

P r o o f. Note that a C-compact subset of X is a bounded set. So by Theorem 4.6 (see [14]), $C_{rc}(X)$ is uniformly complete. □

2. Complete metrizability and some related completeness properties of $C_{rc}(X)$

In this section, we study various kinds of completeness $C_{rc}(X)$. In particular, here we study the complete metrizability of $C_{rc}(X)$ in a wider setting, more precisely, in relation to several other completeness properties.

A space X is called Čech-complete if X is a G_δ-set in βX. A space X is called locally Čech-complete if every point $x \in X$ has a Čech-complete neighborhood. Another completeness property which is implied by Čech-completeness is that of pseudocompleteness.

This is space having a sequence of π-bases $\{\mathcal{B}_n : n \in \mathbb{N}\}$ such that whenever $B_n \in \mathcal{B}_n$ for each n and $\overline{B_{n+1}} \subseteq B_n$, then $\bigcap\{B_n : n \in \mathbb{N}\} \neq \emptyset$ (see [20]).

In [6], it has been shown that a space having a dense Čech-complete subspace is pseudocomplete and a pseudocomplete space is a Baire space.

Let $\mathcal{F}$ and $\mathcal{U}$ be two collections of subsets of X. Then $\mathcal{F}$ is said to be controlled by $\mathcal{U}$, if for each $U \in \mathcal{U}$, there exists $F \in \mathcal{F}$ such that $F \subseteq U$. A sequence $\{U_n\}$ of subsets of X is said to be complete if every filter base $\mathcal{F}$ on X which is controlled $\{U_n\}$ clusters at some $x \in X$. A sequence $\{\mathcal{U}_n\}$ of collections of subsets of X is called complete if $\{U_n\}$ is a complete sequence of subsets of X whenever $U_n \in \mathcal{U}_n$ for all $n \in \mathbb{N}$. It has been shown in [11, Theorem 2.8] that the following statements are equivalent for a Tychonoff space X:

(1) X is a G_δ-subset of any Hausdorff space in which it is densely embedded;

(2) X has a complete sequence of open covers;

(3) X is Čech-complete.

From this result, it easily follows that a Tychonoff space X is Čech-complete if and only if X is a G_δ-subset of any Tychonoff space in which it is densely embedded.

We call a $\mathcal{U}$ of subsets of X an almost-cover of X if $\bigcup \mathcal{U}$ is dense in X. We call a space almost Čech-complete if X has a complete sequence of open almost-covers. Every almost Čech-complete space is a Baire space, see [16, Proposition 4.5].

The property of being a Baire space is the weakest one among the completeness properties we consider here. Since $C_{rc}(X)$ is a locally convex space, $C_{rc}(X)$ is a Baire space if and only if $C_{rc}(X)$ is of second category in itself. Also since a locally convex Baire space is barreled, first we find a necessary condition for $C_{rc}(X)$ to be barreled. A locally convex space L is called barreled if each barrel in L is a neighborhood of 0_L.

Theorem 2.1. *If $C_{rc}(X)$ is barreled, then every bounded subset of X is contained in a C-compact subset of X.*

P r o o f. Let A be a bounded subset of X and let $W = \{f \in C(X) : p_A(f) \leq 1\}$. Then it is routine to check that W is closed, convex, balanced and absorbing, that is, W is a barrel in $C_{rc}(X)$. Since $C_{rc}(X)$ is barreled, W is a neighborhood of f_0 and consequently there exist a closed C-compact subset P of X and $\varepsilon > 0$ such that $\langle f_0, P, \varepsilon\rangle \subseteq W$. We claim that $A \subseteq P$. If not, let $x_0 \in A \setminus P$. So there exists a continuous function $f : X \mapsto [0, 2]$ such that $f(x) = 0$ for all $x \in P$ and $f(x_0) = 2$. Clearly $f \in \langle f_0, P, \varepsilon\rangle$, but $f \notin W$. Hence we must have $A \subseteq P$. □

If X is μ-space, then every closed bounded (C-compact) subset of X is a compact and consequently the C-compact-open and compact-open topologies on $C(X)$ coincide. But by famous Nachbin-Shirota theorem, $C_c(X)$ is barreled if X is μ-space. Hence if X is realcompact, then $C_{rc}(X)$ is barreled. In particular, since the Niemytzki plane L is realcompact, $C_{rc}(L)$ is barreled.

But there are space X such that $C_{rc}(X)$ is not barreled.

E x a m p l e 1. (Dieudonné Plank) Let $X = [0, \omega_1] \times [0, \omega_0] \setminus \{(\omega_1, \omega_0)\}$.

Topology τ is generated from the base: all the points of set $[0, \omega_1) \times [0, \omega_0)$ are isolated, and sets of the form $U_\alpha(\beta) = \{(\beta, \gamma) : \alpha < \gamma \leqslant \omega_0\}$ and $V_\alpha(\beta) = \{(\gamma, \beta) : \alpha < \gamma \leqslant \omega_1\}$.

Let $A = \{(\omega_1, n) : 0 \leqslant n < \omega_0\}$. Take an arbitrary C-compact subset B of the space X. Since a set $\{\alpha\} \times [0, \omega_0]$ is a clopen (and hence functionally open) for any $\alpha < \omega_1$, then set $([0, \omega_1] \times \{\beta\}) \bigcap B$ consists of more than finite number of points for any $\beta \leqslant \omega_0$.

It follows that B is a compact subset of X. In [14] was proved that the set A is a closed bounded subset of X. Since A is not compact subset of X and each C-compact subsets of X is a compact then $C_{rc}(X)$ is not barreled.

Recall that a space X is called hemi-C-compact if there exists a sequence of C-compact subsets $\{A_n : n \in \mathbb{N}\}$ in X such that for any C-compact subset A of X, $A \subseteq A_{n_0}$ holds for some $n_0 \in \mathbb{N}$ [3].

In [3] obtained the characterization of metrizability of space $C_{rc}(X)$.

Theorem 2.2. *For any space X, the following are equivalent.*
1. *$C_{rc}(X)$ is metrizable.*
2. *$C_{rc}(X)$ is of first countable.*
3. *$C_{rc}(X)$ is of countable type.*
4. *$C_{rc}(X)$ is of pointwise countable type.*
5. *$C_{rc}(X)$ has a dense subspace of pointwise countable type.*
6. *$C_{rc}(X)$ is an M-space.*
7. *$C_{rc}(X)$ is a q-space.*
8. *X is hemi-C-compact.*

The following theorem gives a characterization of complete metrizable of the space $C_{rc}(X)$.

Theorem 2.3. *For any space X, the following assertions are equivalent.*
1. *$C_{rc}(X)$ is a completely metrizable.*
2. *$C_{rc}(X)$ is Čech-complete.*
3. *$C_{rc}(X)$ is locally Čech-complete.*
4. *$C_{rc}(X)$ is an open continuous image of a paracompact Čech-complete space.*
5. *$C_{rc}(X)$ is an open continuous image of a Čech-complete space.*
6. *X is a hemi-C-compact rc_f-space.*

P r o o f. We have earlier noted that $C_{rc}(X)$ is completely metrizable if and only if it is uniform complete and metrizable. Hence by Theorem 1.1 and by Theorem 2.2 , (1) $\Leftrightarrow$ (6). Note that (1) $\Rightarrow$ (2) $\Rightarrow$ (3) and (1) $\Rightarrow$ (4) $\Rightarrow$ (5). Also (3) $\Rightarrow$ (5), see [5, 3.12.19.(d)].

(5) $\Rightarrow$ (1) A Čech-complete space is of pointwise countable type and the property of being pointwise countable type is preserved by open continuous maps. Hence $C_{rc}(X)$ is of pointwise countable type and consequently by Corollary 5.2 [3], $C_{rc}(X)$ is metrizable and hence $C_{rc}(X)$ is paracompact. So by Pasynkov's theorem [5, Theorem 5.5.8 (b)], $C_{rc}(X)$ is Čech-complete. But a Čech-complete metrizable space is completely metrizable. □

Note that proof of Theorem 2.3 is similar to that of Theorem 3.3 in [15] on a complete metrizable of space $C(X)$ with the pseudocompact-open topology.

For studying the properties of pseudocomplete and almost Čech-complete we need to embed the space $C_{rc}(X)$ in a larger locally convex function space.

Let $RC(X) = \{f \in \mathbb{R}^X : f|_A$ is continuous for each C-compact subset A of $X\}$. As in case of $C_{rc}(X)$, we can define C-compact-open topology on $RC(X)$. In particular, this is is a locally convex Hausdorff topology on $RC(X)$ generated by the family of seminorms $\{p_A : A$ is a C-compact subset of $X\}$, where for $f \in RC(X)$, $p_A(f) = \sup\{|f(x)| : x \in A\}$.

We denote the space $RC(X)$ with the C-compact-open topology by $RC_{rc}(X)$. It is clear that $C_{rc}(X)$ is a subspace of $RC_{rc}(X)$. Moreover the proof of the following result it immediate.

Theorem 2.4. *If every closed C-compact subset of X is C-embedded in X, then $C(X)$ is dense in $RC_{rc}(X)$.*

In next theorem, the term σ-space refers to a space having a σ-locally finite network. Every metrizable space is a σ-space.

Theorem 2.5. *For a space X, consider the following conditions.*
1. *$C_{rc}(X)$ is completely metrizable.*
2. *$C_{rc}(X)$ is a pseudocomplete σ-space.*
3. *$C_{rc}(X)$ is a pseudocomplete q-space.*
4. *$C_{rc}(X)$ contains a dense completely metrizable subspace.*
5. *$C_{rc}(X)$ contains a dense Čech-complete subspace.*
6. *$C_{rc}(X)$ is almost Čech-complete.*

Then (1) ⇒ (2) ⇒ (3) ⇒ (4) ⇒ (5) ⇔ (6)

P r o o f. (1) ⇒ (2) and (4) ⇒ (5). These are immediate.

(2) ⇒ (3). A Baire space, which is a σ-space as well, has a dense metrizable subspace, see [43]. So if $C_{rc}(X)$ is a pseudocomplete σ-space, then it contains a dense metrizable space. Since every metrizable space is of pointwize countable type, by Theorem 2.2, $C_{rc}(X)$ is a q-space.

(3) ⇒ (4) If $C_{rc}(X)$ is a q-space, then by Theorem 2.2, $C_{rc}(X)$ is metrizable. But a metrizable space is pseudocomplete if and only if it contains a dense completely metrizable subspace, see [6, Corollary 2.4.].

(5) ⇔ (6) follows from [16, Propositions 4.4, 4.7]. □

R e m a r k. If $C_{rc}(X)$ is only assumed to be pseudocomplete, it may not be almost Čech-complete. Let S be an uncountable space in which all points are isolated except for a distinguished point s, a neighborhood of s being any set containing s whose complement is countable [12, 4N].

It can be easily shown that S is a normal space. Since every C-compact subset of S is finite, the C-compact-open topology on $C(S)$ coincides with the point-open topology on $C(S)$. Since S is uncountable, $C_p(S)$ is not metrizable. But $C_p(S)$ is pseudocomplete, since every countable subset in a P-space is closed. But since $C_p(X)$ is not metrizable, by Theorem 5.7 [13], it is not almost Čech-complete either.

Theorem 2.6. *If every closed C-compact subset X is C-embedded in X, then the following assertions are equivalent.*
1. *$C_{rc}(X)$ is completely metrizable.*
2. *$C_{rc}(X)$ is almost Čech-complete.*
3. *X is a hemi-C-compact rc_f-space.*

P r o o f. We only need to show that (2) ⇒ (3). If $C_{rc}(X)$ is almost Čech-complete, then $C_{rc}(X)$ contains a dense Čech-complete subspace G. Since every closed C-compact subset of X is C-embedded in X, $C(X)$ is dense in $RC_{rc}(X)$ and consequently G is dense in $RC_{rc}(X)$. Now since $RC_{rc}(X)$ contains a dense Baire subspace G, $RC_{rc}(X)$ is itself a Baire space. Also since G is Čech-complete, G is a G_δ-set in $RC_{rc}(X)$.

Note that every rc-continuous function on X is in $RC_{rc}(X)$. In order to show that X is a rc_f-space, we will show that $RC(X) = C(X)$. So let $f \in RC(X)$. Define the map $T_f : RC_{rc}(X) \mapsto$

$RC_{rc}(X)$ by $T_f(g) = f + g$ for all $g \in RC(X)$. Since $RC_{rc}(X)$ is a locally convex space, T_f is a homeomorphism and consequently $T_f(G)$ is a dense G_δ-subset of $RC_{rc}(X)$. Since $RC_{rc}(X)$ is a Baire space, $G \bigcap T_f(G) \neq \emptyset$. Let $h \in G \bigcap T_f(G)$. Then there exists $g \in G$ such that $h = f + g$. So $f = g - h \in C(X)$. □

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