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# A CHARACTERIZATION OF DERIVATIONS AND AUTOMORPHISMS ON SOME SIMPLE ALGEBRAS

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**Abstract:** In the present paper, we study simple algebras, which do not belong to the well-known classes of algebras (associative algebras, alternative algebras, Lie algebras, Jordan algebras, etc.). The simple finite-dimensional algebras over a field of characteristic 0 without finite basis of identities, constructed by Kislitsin, are such algebras. In the present paper, we consider two such algebras: the simple seven-dimensional anticommutative algebra $\mathcal{D}$ and the seven-dimensional central simple commutative algebra $\mathcal{C}$. We prove that every local derivation of these algebras $\mathcal{D}$ and $\mathcal{C}$ is a derivation, and every 2-local derivation of these algebras $\mathcal{D}$ and $\mathcal{C}$ is also a derivation. We also prove that every local automorphism of these algebras $\mathcal{D}$ and $\mathcal{C}$ is an automorphism, and every 2-local automorphism of these algebras $\mathcal{D}$ and $\mathcal{C}$ is also an automorphism.

**Keywords:** Simple algebra, Derivation, Local derivation, 2-Local derivation, Automorphism, Local automorphism, 2-Local automorphism, Basis of identities.

## 1. Introduction

In the present paper, we study local and 2-local derivations and automorphisms of simple finite-dimensional algebras without finite basis of identities, constructed by Kislitsin in [19] and [20]. Kadison in [12] introduced and investigated a notion of local derivations. He proved that each continuous local derivation from a von Neumann algebra into its dual Banach bimodule is a derivation. Šemrl introduced a similar notion of 2-local derivations. He proved that any 2-local derivation of the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space $H$ is a derivation [24]. After, numerous new results related to the description of local and 2-local derivations of associative algebras have appeared. For example, papers [1, 3, 4, 15, 16, 22] are devoted to local and 2-local derivations of associative algebras.

The study of local and 2-local derivations of nonassociative algebras was initiated in papers [5, 6] of Ayupov and Kudaybergenov (for the case of Lie algebras). They proved that each local and 2-local derivation on a semisimple finite-dimensional Lie algebra are derivations. In [8], examples of 2-local derivations on nilpotent Lie algebras that are not derivations are given. After the cited works, the study of local and 2-local derivations was continued for Leibniz algebras [7] and Jordan algebras [2]. Local and 2-local automorphisms were also studied in many cases. For example, local and 2-local automorphisms on Lie algebras have been studied in [5, 10].

The variety of Malcev algebras is a generalization of the variety of Lie algebras [23]. It is closely related to other classes of nonassociative structures: it is a proper subvariety of binary Lie algebras, and, under the multiplication $ab - ba$, an alternative algebra is a Malcev algebra. Moreover, it is connected with various classes of algebraic systems such as Moufang loops, Poisson–Malcev algebras, etc. The study of generalizations of derivations of simple Malcev algebras was initiated by Filippov in [11] and continued in some papers of Kaygorodov and Popov [13, 14].

Now, a linear operator $\nabla$ on $\mathcal{A}$ is called a local automorphism if, for every $x \in \mathcal{A}$, there exists an automorphism $\phi_x$ of $\mathcal{A}$, depending on $x$, such that $\nabla(x) = \phi_x(x)$. The concept of local automorphism was introduced by Larson and Sourour [21] in 1990. They proved that invertible local automorphisms of the algebra of all bounded linear operators on an infinite-dimensional Banach space $X$ are automorphisms.

A similar notion, which characterizes non-linear generalizations of automorphisms, was introduced by Šemrl in [24] as 2-local automorphisms. Namely, a map $\Delta : \mathcal{A} \to \mathcal{A}$ (not necessarily linear) is called a 2-local automorphism if, for every $x, y \in \mathcal{A}$, there exists an automorphism $\phi_{x,y} : \mathcal{A} \to \mathcal{A}$ such that $\Delta(x) = \phi_{x,y}(x)$ and $\Delta(y) = \phi_{x,y}(y)$. After the work of Šemrl, it appeared numerous new results related to the description of local and 2-local automorphisms of algebras (see, for example, [5, 7, 9, 10, 16]).

In the present paper, we continue the study of derivations and automorphisms of simple algebras. We study derivations and automorphisms of simple algebras, which do not belong to well-known classes of algebras (commutative, associative, alternative, Lie, Jordan, etc.). The simple finite-dimensional algebras without finite basis of identities, constructed by Kislitsin are such algebras. Namely, we prove that any local derivation (automorphism) of the simple finite-dimensional algebras without finite basis of identities, constructed by Kislitsin in [19] and [20], is a derivation (an automorphism, respectively), and every 2-local derivation (automorphism) of these algebras is also a derivation (an automorphism, respectively). Note that central simple finite-dimensional algebras which has no finite basis of identities were considered in the works [17] and [18] of Isaev and Kislitsin.

## 2. A simple finite-dimensional algebra without finite basis of identities

Let $\mathcal{D} = \langle e, v_1, v_2, e_{11}, e_{12}, e_{22}, p\rangle_{\mathbb{F}}$ be an algebra over a field $\mathbb{F}$ of characteristic 0 whose nonzero products of basis elements from

$$\{e, v_1, v_2, e_{11}, e_{12}, e_{22}, p\} \tag{2.1}$$

are defined by the rules

$$v_ie_{ij} = -e_{ij}v_i = v_j, \quad v_2p = -pv_2 = e, \quad v_ie = -ev_i = v_i,$$
$$e_{ij}e = -ee_{ij} = e_{ij}, \quad pe = -ep = p.$$

Then $\mathcal{D}$ is a simple anticommutative algebra without finite basis of identities [20]. Let $a$ be an element in $\mathcal{D}$. Then we can write

$$a = a_1e + a_2v_1 + a_3v_2 + a_4e_{11} + a_5e_{12} + a_6e_{22} + a_7p$$

for some elements $a_1, a_2, a_3, a_4, a_5, a_6$, and $a_7$ in $\mathbb{F}$. Throughout the paper, let

$$\overline{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)^T.$$

Conversely, if $v = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)^T$ is a column vector with $a_1, a_2, a_3, a_4, a_5, a_6$, and $a_7$ in $\mathbb{F}$, then, throughout the paper, we will denote by $\widehat{v}$ the element

$$a_1 e + a_2 v_1 + a_3 v_2 + a_4 e_{11} + a_5 e_{12} + a_6 e_{22} + a_7 p;$$

i.e.,

$$\widehat{v} = a_1 e + a_2 v_1 + a_3 v_2 + a_4 e_{11} + a_5 e_{12} + a_6 e_{22} + a_7 p.$$

Let $\mathcal{A}$ be an algebra. A linear map $D\colon \mathcal{A} \to \mathcal{A}$ is called a derivation if

$$D(xy) = D(x)y + xD(y)$$

for any two elements $x, y \in \mathcal{A}$.

Our principal tool for the description of local and 2-local derivations of $\mathcal{D}$ is the following proposition.

**Proposition 1.** *A linear map $D\colon \mathcal{D} \to \mathcal{D}$ is a derivation if and only if the matrix of $D$ in the standard basis* (2.1) *has the following form*:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2} + a_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(a_{2,2} + a_{5,5}) \end{pmatrix}.$$

*Here the action of $D$ corresponds to multiplying the matrix by a column on the right.*

P r o o f. The proof is carried out by checking the derivation property on the algebra $\mathcal{D}$.

Let $A = (a_{i,j})_{i,j=1}^7$ be the matrix of the derivation $D$. Then

$$A\overline{v_i e_{ij}} = -A\overline{e_{ij} v_i} = A\overline{v_j}, \quad A\overline{v_2 p} = -A\overline{p v_2} = A\overline{e}, \quad A\overline{v_i e} = -A\overline{e v_i} = A\overline{v_i},$$
$$A\overline{e_{ij} e} = -A\overline{e e_{ij}} = A\overline{e_{ij}}, \quad A\overline{pe} = -A\overline{ep} = A\overline{p}.$$

On the other hand,

$$\widehat{A\overline{v_i e_{ij}}} = \widehat{A\overline{v_i}} e_{ij} + v_i \widehat{A\overline{e_{ij}}}.$$

Hence,

$$\widehat{A\overline{v_j}} = \widehat{A\overline{v_i}} e_{ij} + v_i \widehat{A\overline{e_{ij}}}.$$

So,

$$\widehat{A\overline{v_1}} = \widehat{A\overline{v_1}} e_{11} + v_1 \widehat{A\overline{e_{11}}},$$
$$a_{1,2} e + a_{2,2} v_1 + a_{3,2} v_2 + a_{4,2} e_{11} + a_{5,2} e_{12} + a_{6,2} e_{22} + a_{7,2} p$$
$$= (a_{1,2} e + a_{2,2} v_1 + a_{3,2} v_2 + a_{4,2} e_{11} + a_{5,2} e_{12} + a_{6,2} e_{22} + a_{7,2} p) e_{11}$$
$$+ v_1 (a_{1,4} e + a_{2,4} v_1 + a_{3,4} v_2 + a_{4,4} e_{11} + a_{5,4} e_{12} + a_{6,4} e_{22} + a_{7,4} p)$$

if $i = 1$, $j = 1$, and

$$\begin{aligned} &a_{1,2}e + a_{2,2}v_1 + a_{3,2}v_2 + a_{4,2}e_{11} + a_{5,2}e_{12} + a_{6,2}e_{22} + a_{7,2}p \\ &= -a_{1,2}e_{11} + a_{2,2}v_1 + a_{1,4}v_1 + a_{4,4}v_1 + a_{5,4}v_2. \end{aligned}$$

This implies that

$$a_{1,2} = 0, \quad a_{1,4} + a_{4,4} = 0, \quad a_{3,2} = a_{5,4}, \quad a_{4,2} = -a_{1,2} = 0, \quad a_{5,2} = 0, \quad a_{6,2} = 0, \quad a_{7,2} = 0.$$

In addition, if $i = 1$ and $j = 2$, then

$$\widehat{A\overline{v_2}} = \widehat{A\overline{v_1}}e_{12} + v_1\widehat{A\overline{e_{12}}}$$

and

$$\begin{aligned} &a_{1,3}e + a_{2,3}v_1 + a_{3,3}v_2 + a_{4,3}e_{11} + a_{5,3}e_{12} + a_{6,3}e_{22} + a_{7,3}p \\ &= (a_{1,2}e + a_{2,2}v_1 + a_{3,2}v_2 + a_{4,2}e_{11} + a_{5,2}e_{12} + a_{6,2}e_{22} + a_{7,2}p)e_{12} \\ &+v_1(a_{1,5}e + a_{2,5}v_1 + a_{3,5}v_2 + a_{4,5}e_{11} + a_{5,5}e_{12} + a_{6,5}e_{22} + a_{7,5}p) \\ &= -a_{1,2}e_{12} + a_{2,2}v_2 + a_{1,5}v_1 + a_{4,5}v_1 + a_{5,5}v_2. \end{aligned}$$

This implies that

$$\begin{gathered} a_{1,3} = 0, \quad a_{2,3} = a_{1,5} + a_{4,5}, \quad a_{3,3} = a_{2,2} + a_{5,5}, \\ a_{4,3} = 0, \quad a_{5,3} = -a_{1,2}, \quad a_{6,3} = 0, \quad a_{7,3} = 0. \end{gathered}$$

Besides, if $i = 2$ and $j = 2$, then

$$\widehat{A\overline{v_2}} = \widehat{A\overline{v_2}}e_{22} + v_2\widehat{A\overline{e_{22}}}$$

and

$$\begin{aligned} &a_{1,3}e + a_{2,3}v_1 + a_{3,3}v_2 + a_{4,3}e_{11} + a_{5,3}e_{12} + a_{6,3}e_{22} + a_{7,3}p \\ &= (a_{1,3}e + a_{2,3}v_1 + a_{3,3}v_2 + a_{4,3}e_{11} + a_{5,3}e_{12} + a_{6,3}e_{22} + a_{7,3}p)e_{22} \\ &+v_2(a_{1,6}e + a_{2,6}v_1 + a_{3,6}v_2 + a_{4,6}e_{11} + a_{5,6}e_{12} + a_{6,6}e_{22} + a_{7,6}p) \\ &= -a_{1,3}e_{22} + a_{3,3}v_2 + a_{1,6}v_2 + a_{6,6}v_2 + a_{7,6}e. \end{aligned}$$

This implies that

$$\begin{gathered} a_{1,3} = a_{7,6} = 0, \quad a_{2,3} = 0, \quad a_{1,6} + a_{6,6} = 0, \quad a_{4,3} = 0, \\ a_{5,3} = 0, \quad a_{6,3} = -a_{1,3} = 0, \quad a_{7,3} = 0. \end{gathered}$$

Similarly, we have

$$\begin{gathered} a_{3,3} = -a_{7,7}, \quad a_{2,1} = 0, \quad a_{1,7} = 0, \quad a_{6,7} = 0, \quad a_{4,1} = 0, \quad a_{5,1} = 0, \quad a_{6,1} = 0, \\ a_{1,2} = 0, \quad a_{4,1} = 0, \quad a_{5,1} = 0, \quad a_{1,3} = 0, \quad a_{7,1} = 0, \quad a_{6,1} = 0, \\ a_{1,4} = 0, \quad a_{2,1} = 0, \quad a_{1,1} = 0, \quad a_{1,5} = 0, \quad a_{1,6} = 0, \quad a_{3,1} = 0, \quad a_{1,7} = 0, \\ a_{6,2} = 0, \quad a_{7,2} = 0, \quad a_{4,3} = 0, \quad a_{3,6} = 0, \quad a_{3,7} = 0, \quad a_{3,5} = 0, \quad a_{2,7} = 0, \\ a_{3,4} = 0, \quad a_{1,2} = 0, \quad a_{3,2} = 0, \quad a_{4,7} = 0, \quad a_{5,7} = 0, \quad a_{2,6} = 0, \quad a_{3,4} = 0, \quad a_{2,4} = 0, \\ a_{2,5} = 0, \quad a_{1,2} = 0, \quad a_{5,6} = 0, \quad a_{4,6} = 0, \quad a_{1,3} = 0, \quad a_{2,3} = 0, \quad a_{6,5} = 0, \quad a_{7,5} = 0, \\ a_{6,4} = 0, \quad a_{7,4} = 0. \end{gathered}$$

As a result, we get the matrix from Proposition 1. The proof is complete. □

Let $\mathcal{A}$ be an algebra. A linear map $\nabla\colon \mathcal{A} \to \mathcal{A}$ is called a local derivation if, for any element $x \in \mathcal{A}$, there exists a derivation $D\colon \mathcal{A} \to \mathcal{A}$ such that $\nabla(x) = D(x)$.

**Theorem 1.** *Each local derivation on the simple algebra $\mathcal{D}$ is a derivation.*

P r o o f. Let $\nabla$ be a local derivation on $\mathcal{D}$, and let $A = (a_{i,j})_{i,j=1}^7$ be the matrix of $\nabla$. Then

$$\nabla(v_1) = a_{2,2}^{v_1} v_1 = a_{2,2} v_1, \quad \nabla(v_2) = (a_{2,2}^{v_2} + a_{5,5}^{v_2}) v_2 = a_{3,3} v_2,$$
$$\nabla(e_{1,2}) = a_{5,5}^{e_{1,2}} e_{1,2} = a_{5,5} e_{1,2}, \quad \nabla(p) = -(a_{2,2}^{p} + a_{5,5}^{p}) p = a_{7,7} p,$$

and the remaining components of the matrix $A$ are equal to zero. At the same time,

$$\nabla(v_1 + v_2 + e_{1,2} + p) = \nabla(v_1) + \nabla(v_2) + \nabla(e_{1,2}) + \nabla(p) \tag{2.2}$$

and

$$\nabla(v_1 + v_2 + e_{1,2} + p) = a_{2,2}^{v_1+v_2+e_{1,2}+p} v_1 + (a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p}) v_2$$
$$+ a_{5,5}^{v_1+v_2+e_{1,2}+p} e_{1,2} - (a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p}) p.$$

By 2.2, we have

$$a_{2,2}^{v_1+v_2+e_{1,2}+p} v_1 + (a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p}) v_2$$
$$+ a_{5,5}^{v_1+v_2+e_{1,2}+p} e_{1,2} - (a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p}) p$$
$$= a_{2,2}^{v_1} v_1 + (a_{2,2}^{v_2} + a_{5,5}^{v_2}) v_2 + a_{5,5}^{e_{1,2}} e_{1,2} - (a_{2,2}^{p} + a_{5,5}^{p}) p.$$

Hence,

$$a_{2,2}^{v_1+v_2+e_{1,2}+p} = a_{2,2}^{v_1}, \quad a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p} = a_{2,2}^{v_2} + a_{5,5}^{v_2},$$
$$a_{5,5}^{v_1+v_2+e_{1,2}+p} = a_{5,5}^{e_{1,2}}, \quad a_{2,2}^{v_1+v_2+e_{1,2}+p} + a_{5,5}^{v_1+v_2+e_{1,2}+p} = a_{2,2}^{p} + a_{5,5}^{p}.$$

This implies that

$$a_{2,2}^{v_2} + a_{5,5}^{v_2} = a_{2,2}^{v_1} + a_{5,5}^{e_{1,2}}, \quad a_{2,2}^{p} + a_{5,5}^{p} = a_{2,2}^{v_1} + a_{5,5}^{e_{1,2}}$$

and

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2}^{v_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}^{v_1} + a_{5,5}^{e_{1,2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5}^{e_{1,2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(a_{2,2}^{v_1} + a_{5,5}^{e_{1,2}}) \end{pmatrix}.$$

Hence, by Proposition 1, $\nabla$ is a derivation. This completes the proof. □

We give another characterization of derivations on the algebra $\mathcal{D}$ in the following theorem.

Let $\mathcal{A}$ be an algebra. A (not necessary linear) map $\Delta : \mathcal{A} \to \mathcal{A}$ is called a 2-local derivation if, for all elements $x, y \in \mathcal{A}$, there exists a derivation $D_{x,y} : \mathcal{A} \to \mathcal{A}$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$.

**Theorem 2.** *Each 2-local derivation on the simple algebra $\mathcal{D}$ is a derivation.*

P r o o f. Suppose that $\Delta$ is a 2-local derivation on $\mathcal{D}$ and, for elements $a, b \in \mathcal{D}$, $D_{a,b}$ is a derivation on $\mathcal{D}$ such that $D_{a,b}(a) = \Delta(a)$ and $D_{a,b}(b) = \Delta(b)$. Let $A_{a,b} = (a_{i,j}^{a,b})_{i,j=1}^{7}$ be the matrix of $D_{a,b}$.

Let

$$a = \lambda_1 e + \lambda_2 v_1 + \lambda_3 v_2 + \lambda_4 e_{1,1} + \lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \lambda_7 p$$

be an arbitrary element from $\mathcal{D}$. For every $v \in \mathcal{D}$, there exists a derivation $D_{v,a}$ such that

$$\Delta(v) = D_{v,a}(v), \quad \Delta(a) = D_{v,a}(a).$$

Then from

$$D_{v_1,v}(v_1) = D_{v_1,a}(v_1), \quad v \in \mathcal{D},$$

it follows that

$$a_{2,2}^{v_1,v} v_1 = a_{2,2}^{v_1,a} v_1.$$

Hence,

$$a_{2,2}^{v_1,v} = a_{2,2}^{v_1,a}.$$

Therefore,

$$\Delta(a) = D_{v_1,a}(a) = a_{2,2}^{v_1,v}\lambda_2 v_1 + (a_{2,2}^{v_1,a} + a_{5,5}^{v_1,a})\lambda_3 v_2 + a_{5,5}^{v_1,a}\lambda_5 e_{1,2} - (a_{2,2}^{v_1,a} + a_{5,5}^{v_1,a})\lambda_7 p.$$

Similarly, from

$$D_{v_2,v}(v_2) = D_{v_2,a}(v_2), \quad v \in \mathcal{D},$$

it follows that

$$\Delta(a) = D_{v_2,a}(a) = a_{2,2}^{v_2,a}\lambda_2 v_1 + (a_{2,2}^{v_2,v} + a_{5,5}^{v_2,v})\lambda_3 v_2 + a_{5,5}^{v_2,a}\lambda_5 e_{1,2} - (a_{2,2}^{v_2,a} + a_{5,5}^{v_2,a})\lambda_7 p.$$

Similarly, we have

$$\Delta(a) = D_{e_{1,2},a}(a) = a_{2,2}^{e_{1,2},a}\lambda_2 v_1 + (a_{2,2}^{e_{1,2},a} + a_{5,5}^{e_{1,2},a})\lambda_3 v_2 + a_{5,5}^{e_{1,2},v}\lambda_5 e_{1,2} - (a_{2,2}^{e_{1,2},a} + a_{5,5}^{e_{1,2},a})\lambda_7 p,$$
$$\Delta(a) = D_{p,a}(a) = a_{2,2}^{p,a}\lambda_2 v_1 + (a_{2,2}^{p,a} + a_{5,5}^{p,a})\lambda_3 v_2 + a_{5,5}^{p,a}\lambda_5 e_{1,2} - (a_{2,2}^{p,v} + a_{5,5}^{p,v})\lambda_7 p.$$

Hence,

$$\Delta(a) = D_{v_1,a}(a) = D_{v_2,a}(a) = D_{e_{1,2},a}(a) = D_{p,a}(a) =$$
$$a_{2,2}^{v_1,v}\lambda_2 v_1 + (a_{2,2}^{v_2,w} + a_{5,5}^{v_2,w})\lambda_3 v_2 + a_{5,5}^{e_{1,2},z}\lambda_5 e_{1,2} - (a_{2,2}^{p,t} + a_{5,5}^{p,t})\lambda_7 p$$

for any $v, w, z, t \in \mathcal{D}$. Note that the components in the last sum do not depend on the element $a$. Therefore, the map $\Delta$ is linear and it is a local derivation. The linear operator $\Delta$ has the following matrix:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2}^{v_1,v} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}^{v_2,w} + a_{5,5}^{v_2,w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5}^{e_{1,2},z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(a_{2,2}^{p,t} + a_{5,5}^{p,t}) \end{pmatrix}.$$

From $\Delta(v_2+p)=\Delta(v_2)+\Delta(p)$, we get

$$(a_{2,2}^{a,v_2+p}+a_{5,5}^{a,v_2+p})v_2-(a_{2,2}^{a,v_2+p}+a_{5,5}^{a,v_2+p})p=(a_{2,2}^{v_2,w}+a_{5,5}^{v_2,w})v_2-(a_{2,2}^{p,t}+a_{5,5}^{p,t})p.$$

Hence,

$$a_{2,2}^{a,v_2+p}+a_{5,5}^{a,v_2+p}=a_{2,2}^{v_2,w}+a_{5,5}^{v_2,w}=a_{2,2}^{p,t}+a_{5,5}^{p,t}. \tag{2.3}$$

From $\Delta(v_1+v_2+e_{1,2})=\Delta(v_1)+\Delta(v_2)+\Delta(e_{1,2})$, we get

$$\begin{gathered}a_{2,2}^{a,v_1+v_2+e_{1,2}}=a_{2,2}^{v_1,v},\\ a_{2,2}^{a,v_1+v_2+e_{1,2}}+a_{5,5}^{a,v_1+v_2+e_{1,2}}=a_{2,2}^{v_2,w}+a_{5,5}^{v_2,w},\\ a_{5,5}^{a,v_1+v_2+e_{1,2}}=a_{5,5}^{e_{1,2},z}.\end{gathered}$$

Hence,

$$a_{2,2}^{v_2,w}+a_{5,5}^{v_2,w}=a_{2,2}^{v_1,v}+a_{5,5}^{e_{1,2},z}.$$

By (2.3), we also have

$$a_{2,2}^{p,t}+a_{5,5}^{p,t}=a_{2,2}^{v_1,v}+a_{5,5}^{e_{1,2},z}.$$

Thus,

$$A=\begin{pmatrix}0&0&0&0&0&0&0\\ 0&a_{2,2}^{v_1,v}&0&0&0&0&0\\ 0&0&a_{2,2}^{v_1,v}+a_{5,5}^{e_{1,2},z}&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&a_{5,5}^{e_{1,2},z}&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-(a_{2,2}^{v_1,v}+a_{5,5}^{e_{1,2},z})\end{pmatrix}.$$

Therefore, by Proposition 1, $\Delta$ is a derivation. This completes the proof. □

Let $\mathcal{A}$ be an algebra. A linear bijective map $\Phi\colon\mathcal{A}\to\mathcal{A}$ is called an automorphism if $\Phi(xy)=\Phi(x)\Phi(y)$ for any two elements $x,y\in\mathcal{A}$.

Our principal tool for the description of local and 2-local automorphisms of $\mathcal{D}$ is the following proposition.

**Proposition 2.** *A linear map $\Phi\colon\mathcal{D}\to\mathcal{D}$ is an automorphism if and only if the matrix of $\Phi$ in the standard basis (2.1) has the following form:*

$$\begin{pmatrix}1&0&0&0&0&0&0\\ 0&a_{2,2}&0&0&0&0&0\\ 0&0&a_{2,2}a_{5,5}&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&a_{5,5}&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&\frac{1}{a_{2,2}a_{5,5}}\end{pmatrix},$$

*where $a_{2,2}$ and $a_{5,5}$ are nonzero elements from $\mathbb{F}$. Here the action of $\Phi$ corresponds to multiplying the matrix by a column on the right.*

P r o o f. Let $B = (b_{i,j})_{i,j=1}^7$ be the matrix of the automorphism $\Phi$. Then there exists a derivation $D$ such that

$$B = e^A,$$

where $A$ is the matrix of $D$. It is known that

$$e^A = E + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots,$$

where $E$ is the unit matrix. Hence,

$$B = E + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots. \tag{2.4}$$

By (2.4) and Proposition 1, $B$ is equal to

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{i=0}^{\infty} \frac{a_{2,2}^i}{i!} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=0}^{\infty} \frac{(a_{2,2}+a_{5,5})^i}{i!} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{i=0}^{\infty} \frac{a_{5,5}^i}{i!} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sum_{i=0}^{\infty} \frac{(-1)^i(a_{2,2}+a_{5,5})^i}{i!} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{a_{2,2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{a_{2,2}+a_{5,5}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{a_{5,5}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{-(a_{2,2}+a_{5,5})} \end{pmatrix}.$$

The latter matrix gives the desired form. This completes the proof. □

Let $\mathcal{A}$ be an algebra. A linear map $\nabla : \mathcal{A} \to \mathcal{A}$ is called a local automorphism if, for every element $x \in \mathcal{A}$, there exists an automorphism $\phi_x : \mathcal{A} \to \mathcal{A}$ such that $\nabla(x) = \phi_x(x)$.

**Theorem 3.** *Each local automorphism on the simple algebra $\mathcal{D}$ is an automorphism.*

P r o o f. Let $\nabla$ be a local automorphism on $\mathcal{D}$, and let $A = (a_{i,j})_{i,j=1}^7$ be the matrix of $\nabla$. Then

$$\nabla(v_1) = a_{2,2}^{v_1} v_1 = a_{2,2} v_1, \quad \nabla(v_2) = a_{2,2}^{v_2} a_{5,5}^{v_2} v_2 = a_{3,3} v_2,$$
$$\nabla(e_{1,2}) = a_{5,5}^{e_{1,2}} e_{1,2} = a_{5,5} e_{1,2}, \quad \nabla(p) = \frac{1}{a_{2,2}^p a_{5,5}^p} p = a_{7,7} p$$

and the remaining components of the matrix $A$ are equal to zero. At the same time,

$$\nabla(v_1 + v_2 + e_{1,2} + p) = \nabla(v_1) + \nabla(v_2) + \nabla(e_{1,2}) + \nabla(p) \tag{2.5}$$

and

$$\nabla(v_1+v_2+e_{1,2}+p)=a_{2,2}^{v_1+v_2+e_{1,2}+p}v_1+a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p}v_2+ \\ a_{5,5}^{v_1+v_2+e_{1,2}+p}e_{1,2}+\frac{1}{a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p}}p.$$

By (2.5), we have

$$a_{2,2}^{v_1+v_2+e_{1,2}+p}v_1+a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p})v_2+a_{5,5}^{v_1+v_2+e_{1,2}+p}e_{1,2}+\frac{1}{a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p}}p \\ =a_{2,2}^{v_1}v_1+a_{2,2}^{v_2}a_{5,5}^{v_2}v_2+a_{5,5}^{e_{1,2}}e_{1,2}+\frac{1}{a_{2,2}^{p}a_{5,5}^{p}}p.$$

Hence,

$$a_{2,2}^{v_1+v_2+e_{1,2}+p}=a_{2,2}^{v_1},\quad a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p}=a_{2,2}^{v_2}a_{5,5}^{v_2}, \\ a_{5,5}^{v_1+v_2+e_{1,2}+p}=a_{5,5}^{e_{1,2}},\quad a_{2,2}^{v_1+v_2+e_{1,2}+p}a_{5,5}^{v_1+v_2+e_{1,2}+p}=a_{2,2}^{p}a_{5,5}^{p}.$$

This implies that

$$a_{2,2}^{v_2}a_{5,5}^{v_2}=a_{2,2}^{v_1}a_{5,5}^{e_{1,2}},\quad a_{2,2}^{p}a_{5,5}^{p}=a_{2,2}^{v_1}a_{5,5}^{e_{1,2}}$$

and

$$A=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2}^{v_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}^{v_1}a_{5,5}^{e_{1,2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5}^{e_{1,2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_{2,2}^{v_1}a_{5,5}^{e_{1,2}}} \end{pmatrix}.$$

Hence, by Proposition 2, $\nabla$ is an automorphism. This completes the proof. □

A (not necessary linear) map $\Delta:\mathcal{A}\to\mathcal{A}$ is called a 2-local automorphism if, for all elements $x,y\in\mathcal{A}$, there exists an automorphism $\phi_{x,y}:\mathcal{A}\to\mathcal{A}$ such that $\Delta(x)=\phi_{x,y}(x)$ and $\Delta(y)=\phi_{x,y}(y)$.

**Theorem 4.** *Each 2-local automorphism on the simple algebra $\mathcal{D}$ is an automorphism.*

P r o o f. Suppose that $\Delta$ is a 2-local automorphism on $\mathcal{D}$ and, for elements $a,b\in\mathcal{D}$, $\Phi_{a,b}$ is an automorphism on $\mathcal{D}$ such that $\Phi_{a,b}(a)=\Delta(a)$ and $\Phi_{a,b}(b)=\Delta(b)$. Let $A_{a,b}=(a_{i,j}^{a,b})_{i,j=1}^7$ be the matrix of $\Phi_{a,b}$. Then, for all $v,z\in\mathcal{D}$, there exists an automorphism $\Phi_{v,z}$ such that

$$\Delta(v)=\Phi_{v,z}(v),\quad \Delta(z)=\Phi_{v,z}(z).$$

Let $A_{v,z}=(a_{i,j}^{v,z})_{i,j=1}^n$ be the matrix of the automorphism $\Phi_{v,z}$.

Let

$$a=\lambda_1 e+\lambda_2 v_1+\lambda_3 v_2+\lambda_4 e_{1,1}+\lambda_5 e_{1,2}+\lambda_6 e_{2,2}+\lambda_7 p$$

be an arbitrary element from $\mathcal{D}$. For every $v\in\mathcal{D}$, there exists an automorphism $\Phi_{v,a}$ such that

$$\Delta(v)=\Phi_{v,a}(v),\quad \Delta(a)=\Phi_{v,a}(a).$$

Then from

$$\Phi_{v_1,v}(v_1) = \Phi_{v_1,a}(v_1), \quad v \in \mathcal{D},$$

it follows that

$$a_{2,2}^{v_1,v} v_1 = a_{2,2}^{v_1,a} v_1.$$

Hence,

$$a_{2,2}^{v_1,v} = a_{2,2}^{v_1,a}.$$

Therefore,

$$\begin{aligned}\Delta(a) = \Phi_{v_1,a}(a) = \lambda_1 e + a_{2,2}^{v_1,v}\lambda_2 v_1 + a_{2,2}^{v_1,a} a_{5,5}^{v_1,a}\lambda_3 v_2 + \lambda_4 e_{1,1} \\ + a_{5,5}^{v_1,a}\lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \frac{1}{a_{2,2}^{v_1,a} a_{5,5}^{v_1,a}}\lambda_7 p.\end{aligned}$$

Similarly, from

$$\Phi_{v_2,v}(v_2) = \Phi_{v_2,a}(v_2), \quad v \in \mathcal{D},$$

it follows that

$$\begin{aligned}\Delta(a) = \Phi_{v_2,a}(a) = \lambda_1 e + a_{2,2}^{v_2,a}\lambda_2 v_1 + a_{2,2}^{v_2,v} a_{5,5}^{v_2,v}\lambda_3 v_2 + \lambda_4 e_{1,1} \\ + a_{5,5}^{v_2,a}\lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \frac{1}{a_{2,2}^{v_2,a} a_{5,5}^{v_2,a}}\lambda_7 p.\end{aligned}$$

Similarly, we have

$$\begin{aligned}\Delta(a) = \Phi_{e_{1,2},a}(a) = +\lambda_1 e + a_{2,2}^{e_{1,2},a}\lambda_2 v_1 + a_{2,2}^{e_{1,2},a} a_{5,5}^{e_{1,2},a}\lambda_3 v_2 \\ + \lambda_4 e_{1,1} + a_{5,5}^{e_{1,2},v}\lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \frac{1}{a_{2,2}^{e_{1,2},a} a_{5,5}^{e_{1,2},a}}\lambda_7 p, \\ \Delta(a) = \Phi_{p,a}(a) = \lambda_1 e + a_{2,2}^{p,a}\lambda_2 v_1 + a_{2,2}^{p,a} a_{5,5}^{p,a}\lambda_3 v_2 \\ + \lambda_4 e_{1,1} + a_{5,5}^{p,a}\lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \frac{1}{a_{2,2}^{p,v} a_{5,5}^{p,v}}\lambda_7 p.\end{aligned}$$

Hence,

$$\begin{aligned}\Delta(a) = \Phi_{v_1,a}(a) = \Phi_{v_2,a}(a) = \Phi_{e_{1,2},a}(a) = \Phi_{p,a}(a) = \\ \lambda_1 e + a_{2,2}^{v_1,v}\lambda_2 v_1 + a_{2,2}^{v_2,w} a_{5,5}^{v_2,w}\lambda_3 v_2 + \lambda_4 e_{1,1} + a_{5,5}^{e_{1,2},z}\lambda_5 e_{1,2} + \lambda_6 e_{2,2} + \frac{1}{a_{2,2}^{p,t} a_{5,5}^{p,t}}\lambda_7 p\end{aligned}$$

for any $v, w, z, t \in \mathcal{D}$. Note that the components in the last sum do not depend on the element $a$. Therefore, the map $\Delta$ is linear and it is a local automorphism. The linear operator $\Delta$ has the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2}^{v_1,v} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}^{v_2,w} a_{5,5}^{v_2,w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5}^{e_{1,2},z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_{2,2}^{p,t} a_{5,5}^{p,t}} \end{pmatrix}.$$

From $\Delta(v_2+p)=\Delta(v_2)+\Delta(p)$, we get

$$a_{2,2}^{a,v_2+p}a_{5,5}^{a,v_2+p}v_2+\frac{1}{a_{2,2}^{a,v_2+p}a_{5,5}^{a,v_2+p}}p=a_{2,2}^{v_2,w}a_{5,5}^{v_2,w}v_2+\frac{1}{a_{2,2}^{p,t}a_{5,5}^{p,t}}p.$$

Hence,

$$a_{2,2}^{a,v_2+p}a_{5,5}^{a,v_2+p}=a_{2,2}^{v_2,w}a_{5,5}^{v_2,w}=a_{2,2}^{p,t}a_{5,5}^{p,t}. \tag{2.6}$$

From $\Delta(v_1+v_2+e_{1,2})=\Delta(v_1)+\Delta(v_2)+\Delta(e_{1,2})$, we get

$$a_{2,2}^{a,v_1+v_2+e_{1,2}}=a_{2,2}^{v_1,v},\quad a_{2,2}^{a,v_1+v_2+e_{1,2}}a_{5,5}^{a,v_1+v_2+e_{1,2}}=a_{2,2}^{v_2,w}a_{5,5}^{v_2,w},$$

$$a_{5,5}^{a,v_1+v_2+e_{1,2}}=a_{5,5}^{e_{1,2},z}.$$

Hence,

$$a_{2,2}^{v_2,w}a_{5,5}^{v_2,w}=a_{2,2}^{v_1,v}a_{5,5}^{e_{1,2},z}.$$

By (2.6), we also have

$$a_{2,2}^{p,t}a_{5,5}^{p,t}=a_{2,2}^{v_1,v}a_{5,5}^{e_{1,2},z}.$$

Thus,

$$A=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & a_{2,2}^{v_1,v} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & a_{2,2}^{v_1,v}a_{5,5}^{e_{1,2},z} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & a_{5,5}^{e_{1,2},z} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_{2,2}^{v_1,v}a_{5,5}^{e_{1,2},z}} \end{pmatrix}.$$

Therefore, by Proposition 2, $\Delta$ is an automorphism. This completes the proof. □

## 3. A simple central commutative algebra with no finite basis of identities

Let $\mathcal{C}=\langle \mathbf{1},v_1,v_2,e_{11},e_{12},e_{22},p\rangle_{\mathbb{F}}$ be an algebra over a field $\mathbb{F}$ of characteristic 0, where $\mathbf{1}$ is unity and nonzero products of basis elements

$$\{\mathbf{1},v_1,v_2,e_{11},e_{12},e_{22},p\} \tag{3.1}$$

other than $\mathbf{1}$ are defined as follows:

$$v_ie_{ij}=e_{ij}v_i=v_j,\quad v_2p=pv_2=\mathbf{1}.$$

Then the algebra $\mathcal{C}$ is a simple central commutative algebra with no finite basis of identities [19]. Let $a$ be an element in $\mathcal{C}$. Then we can write

$$a=a_1e+a_2v_1+a_3v_2+a_4e_{11}+a_5e_{12}+a_6e_{22}+a_7p,$$

for some elements $a_1, a_2, a_3, a_4, a_5, a_6$, and $a_7$ in $\mathbb{F}$. Throughout the paper, let

$$\overline{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)^T.$$

Conversely, if $v = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)^T$ is a column vector with $a_1, a_2, a_3, a_4, a_5, a_6$, and $a_7$ in $\mathbb{F}$, then, throughout the paper, we will denote by $\widehat{v}$ the element

$$a_1 e + a_2 v_1 + a_3 v_2 + a_4 e_{11} + a_5 e_{12} + a_6 e_{22} + a_7 p,$$

i.e.,

$$\widehat{v} = a_1 e + a_2 v_1 + a_3 v_2 + a_4 e_{11} + a_5 e_{12} + a_6 e_{22} + a_7 p.$$

Our principal tool for the description of local and 2-local derivations of $\mathcal{C}$ is the following proposition.

**Proposition 3.** *A linear map $D\colon \mathcal{C} \to \mathcal{C}$ is a derivation if and only if the matrix of $D$ in the basis (3.1) has the following form:*

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}+a_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(a_{2,2}+a_{5,5}) \end{pmatrix}.$$

*Here the action of $D$ corresponds to multiplying the matrix by a column on the right.*

P r o o f. The proof of this proposition is similar to the proof of Proposition 1. □

**Theorem 5.** *Each local (2-local) derivation on the simple algebra $\mathcal{C}$ is a derivation.*

P r o o f. The proof of this theorem is similar to the proofs of Theorems 1 and 2. □

**Proposition 4.** *A linear map $\Phi\colon \mathcal{C} \to \mathcal{C}$ is an automorphism if and only if the matrix of $\Phi$ in the standard basis (3.1) has the following form:*

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{2,2}a_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_{2,2}a_{5,5}} \end{pmatrix},$$

*where $a_{2,2}$ and $a_{5,5}$ are nonzero elements from $\mathbb{F}$. Here the action of $\Phi$ corresponds to multiplying the matrix by a column on the right.*

**Theorem 6.** *Each local (2-local) automorphism on the simple algebra $\mathcal{C}$ is an automorphism.*

P r o o f. The proof of this theorem is similar to the proofs of Theorems 3 and 4. □

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