

# IMPULSE CONTROL OF THE MANIPULATION ROBOT

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**Abstract:** A nonlinear control problem for a manipulation robot is considered. The solvability conditions for the problem are obtained in the class of special impulse controls. To achieve the control goal, the kinetic energy of the manipulation robot is used. When finding analytical formulas for controls, the classical first integrals of Lagrangian mechanics were used. The effectiveness of the proposed algorithm is illustrated by computer simulation.

**Keywords:** Manipulation robot, Impulse controls, First integrals.

## Introduction

The purpose of controlling the manipulation robot is to transfer it from the initial position to the final. The significant nonlinearity of the mathematical model describing the movements of the manipulation robot does not allow the use of the methods of the mathematical control theory directly for the original model. Decomposition methods make it possible to reduce the dimension of the control problem, passing to approximate linear or integrable mathematical control models [1, 2]. The work considers a manipulation robot with three degrees of freedom, imitating the movement of a human hand, described in the monograph [3]. In [4], the problem of controlling the horizontal two-dimensional motion of this robot was studied. The original nonlinear control model was replaced with an integrable controlled model. The Pontryagin maximum principle was used to find controls in the performance problem. The use of special impulse controls in this work allows the kinetic energy of the manipulation robot to be used to reach the final position. This approach preserves the integrability of the original controlled mathematical model and, when solving the problem, uses the classical first integrals of Lagrangian mechanics. The article concludes with the results of numerical simulation of the algorithm.

## 1. Mathematical statement of the problem

A manipulation robot with three degrees of freedom, imitating the movement of a human hand, described in the monograph [3, p. 263]. Figure 1 shows a robot manipulator. The number 1 indicates the base of the robotic arm, 2 is a rack of vertically oriented shaft. This shaft is rigidly connected to the guide beam 4 and hand 5.

Kinetic energy of the manipulation robot is determined by the formula

$$T = \frac{1}{2} (m_1 z'^2 + (J_1 + J_2) \varphi'^2 + m_2 (z'^2 + x'^2 + x^2 \varphi'^2)),$$

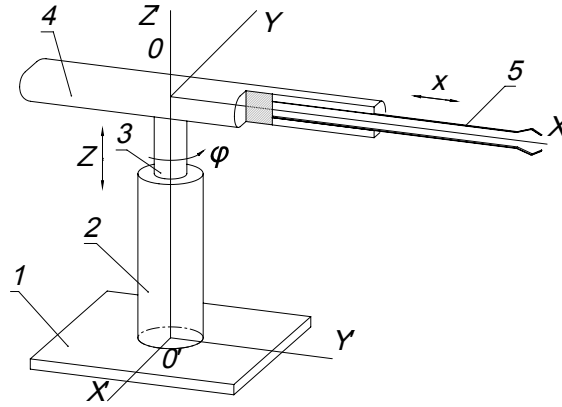


Figure 1

where  $x$  is the coordinate of the center of mass of the arm,  $x > 0$ ;  $z$  is the height of the arm,  $z > 0$ ;  $\varphi$  is the angle of rotation of the hand;  $m_1$  is the total mass of the shaft that rotates the arm, and the guides that implement the horizontal displacement of the arm;  $m_2$  is the mass of the arm;  $J_1$  is the moment of inertia of the shaft and guides relative to the vertical axis;  $J_2$  is the central moment of inertia of the arm relative to the vertical axis. The potential energy of gravity is  $V = (m_1 + m_2)gz$ , where  $g$  is the gravitational acceleration.

The second-order Lagrange equations for the mechanical system under consideration are of the form [3, p. 263]

$$z'' + g = u_1, \quad (1.1)$$

$$x'' - x\varphi'^2 = u_2, \quad (1.2)$$

$$\varphi'' + \alpha^2 (x^2 \varphi')' = u_3. \quad (1.3)$$

Here  $u_1 = P/(m_1 + m_2)$ ,  $u_2 = F/m_2$ ,  $u_3 = M/(J_1 + J_2)$  are the control force actions, where  $P$  is the magnitude of the longitudinal force acting along the vertical axis,  $F$  is the magnitude of the longitudinal force acting along the horizontal guides,  $M$  is the magnitude of the moment of force directed along the vertical axis,  $\alpha = \sqrt{m_2/(J_1 + J_2)}$ .

It is required to find the controls  $u_1$ ,  $u_2$ ,  $u_3$  that translate the system (1.1)–(1.3) from the initial equilibrium position  $(z_0, x_0, \varphi_0)^\top$ ,  $z_0 > 0$ ,  $x_0 > 0$ , to the given end position  $(z_*, x_*, \varphi_*)^\top$ ,  $z_* > 0$ ,  $x_* > 0$ ,  $z_* \neq z_0$ ,  $x_* \neq x_0$ ,  $\varphi_* \neq \varphi_0$ .

To solve this problem, we use a special set  $\mathbb{U}$  of impulse controls defined by the formulas

$$u_1(t) = \dot{z}_0 \delta(t), \quad u_2(t) = \dot{x}_0 \delta(t), \quad u_3(t) = \dot{\varphi}_0 \delta(t), \quad t \in \mathbb{R},$$

where  $\delta(\cdot)$  is the Dirac impulse function. Impulse controls at the initial moment of time  $t = 0$  to the dynamic system (1.1)–(1.3), which are in equilibrium, report the initial velocities  $z'(0) = \dot{z}_0$ ,  $x'(0) = \dot{x}_0$ ,  $\varphi'(0) = \dot{\varphi}_0$ . The initial speeds are control parameters, the choice of which should ensure that the dynamic system (1.1)–(1.3) falls into the final position.

## 2. Impulse control parameters

The problem is divided into two problems. In the first problem, for the equation (1.1) that determines the vertical movement of the manipulation robot, we use the set  $\mathbb{U}_1$  of impulse controls defined by the formula  $u_1(t) = \dot{z}_0 \delta(t)$ ,  $t \in \mathbb{R}$ . It is required to find the control  $u_1$ , which transfers the dynamical system (1.1) from the initial equilibrium position  $z_0$ ,  $z_0 > 0$ , to the given final

position  $z_*$ ,  $z_* > 0$ ,  $z_* \neq z_0$ . In the second problem, for the system of equations (1.2), (1.3), which determines the horizontal movement of the manipulation robot, we use the set  $\mathbb{U}_2$  of impulse controls defined by the formulas  $u_2(t) = \dot{x}_0\delta(t)$ ,  $u_3(t) = \dot{\varphi}_0\delta(t)$ ,  $t \in \mathbb{R}$ . It is required to find controls  $u_2, u_3$  that translate the dynamical system (1.2), (1.3) from the initial equilibrium position  $(x_0, \varphi_0)^\top$ ,  $x_0 > 0$ , to the given end position  $(x_*, \varphi_*)^\top$ ,  $x_* > 0$ ,  $x_* \neq x_0$ ,  $\varphi_* \neq \varphi_0$ .

**Lemma 1.** *In the set  $\mathbb{U}_1$  there are impulse controls  $u_1$  that transfer the dynamical system (1.1) from an arbitrary initial position  $z_0$ ,  $z_0 > 0$ , to an arbitrary end position  $z_*$ ,  $z_* > 0$ ,  $z_* \neq z_0$ .*

**P r o o f.** The impulse control  $u_1 \in \mathbb{U}_1$  provides the dynamic system (1.1) at the initial moment  $t = 0$  the initial velocity  $z'(0) = \dot{z}_0$ . For  $t > 0$ , the motion of a free dynamic system is determined by the differential equation  $z'' + g = 0$  with the initial conditions  $z(0) = z_0$ ,  $z'(0) = \dot{z}_0$ . The vertical movement is described by the formula  $z(t) = -gt^2/2 + \dot{z}_0t + z_0$ ,  $t > 0$ . For any  $\tau_1 > 0$  there is a unique value of the control parameter  $z_0$  for which the equality  $z(\tau_1) = z_*$ ,  $\dot{z}_0 = \tau_1^{-1}(z_* - z_0) + g\tau_1/2$  is true.

We show that the velocity at the finite moment of time is minimal in absolute value if  $\tau_1 = \sqrt{2|z_* - z_0|/g}$ . The velocity at the finite moment of time is determined by the formula

$$z'(\tau_1) = -g\tau_1 + \dot{z}_0 = -g\tau_1/2 + \tau_1^{-1}(z_* - z_0).$$

If  $z_* > z_0 > 0$ , then we have  $z'(\tau_1) = 0$  under the condition  $\tau_1 = \sqrt{2(z_* - z_0)/g}$ . The function  $f(\tau) = g_1/2 + \tau^{-1}(z_0 - z_*)$  has a minimum at  $\tau = \tau_1 = \sqrt{2(z_0 - z_*)/g}$  which is equal to  $\sqrt{2g(z_0 - z_*)}$  if  $z_0 > z_* > 0$ .

The impulse controls  $u_2, u_3 \in \mathbb{U}_2$  prescribe the dynamic system (1.2), (1.3) at the initial moment  $t = 0$  the initial speeds  $x'(0) = \dot{x}_0$ ,  $\varphi'(0) = \dot{\varphi}_0$ . For  $t > 0$  the motion of a free dynamic system is determined by differential equations  $x'' - x\varphi'^2 = 0$ ,  $\varphi'' + \alpha^2(x^2\varphi')' = 0$  with initial conditions  $x(0) = x_0$ ,  $\varphi(0) = \varphi_0$ ,  $x'(0) = \dot{x}_0$ ,  $\varphi'(0) = \dot{\varphi}_0$ . For the horizontal free movement of the manipulation robot, kinetic energy and momentum are kept the same [5]

$$T_2 = \frac{1}{2} ((J_1 + J_2)\varphi'^2 + m_2(x'^2 + x^2\varphi'^2)) = \text{const},$$

$$p_2 = \frac{\partial T_2}{\partial \varphi'} = (J_1 + J_2 + m_2x^2)\varphi' = \text{const}.$$

When describing horizontal motion for  $t > 0$ , we replace the system (1.1), (1.2) with the system of differential equations

$$\alpha^2 x'^2 + (1 + \alpha^2 x^2)\varphi'^2 = c_1, \quad (2.1)$$

$$(1 + \alpha^2 x^2)\varphi' = c_2, \quad (2.2)$$

where

$$c_1 = \alpha^2 \dot{x}_0^2 + (1 + \alpha^2 x_0^2)\dot{\varphi}_0^2, \quad c_2 = (1 + \alpha^2 x_0^2)\dot{\varphi}_0.$$

□

**Lemma 2.** *Let the conditions*

$$|\varphi_* - \varphi_0| \leq \int_{x_0}^{x_*} \frac{\sqrt{1 + \alpha^2 x_0^2} ds}{\sqrt{(1 + \alpha^2 s^2)(s^2 - x_0^2)}}, \quad 0 < x_0 < x_*, \quad (2.3)$$

$$|\varphi_* - \varphi_0| \leq \int_{x_*}^{x_0} \frac{\sqrt{1 + \alpha^2 x_*^2} ds}{\sqrt{(1 + \alpha^2 s^2)(s^2 - x_*^2)}}, \quad 0 < x_* < x_0, \quad (2.4)$$

hold. Then in the set  $\mathbb{U}_2$  there are impulse controls  $u_1, u_2$  that move the dynamical system (1.2), (1.3) from the starting position  $(x_0, \varphi_0)^\top$ ,  $x_0 > 0$ , to the ending position  $(x_*, \varphi_*)^\top$ ,  $x_* > 0$ ,  $x_* \neq x_0$ ,  $\varphi_* \neq \varphi_0$ .

*P r o o f.* Under the condition  $\dot{x}_0 \neq 0$ , the system of differential equations (2.1), (2.2) is transformed to the following form

$$x' = \operatorname{sgn} \dot{x}_0 \sqrt{\dot{x}_0^2 + (1 + \alpha^2 x_0^2) \dot{\varphi}_0^2} \frac{x^2 - x_0^2}{1 + \alpha^2 x^2}, \quad x \in \mathbb{X}, \quad (2.5)$$

$$\varphi' = \frac{(1 + \alpha^2 x_0^2) \dot{\varphi}_0}{1 + \alpha^2 x^2}, \quad x \in \mathbb{X}, \quad (2.6)$$

where

$$\mathbb{X} = \left\{ x \in \mathbb{R}^+ : \dot{x}_0^2 + (1 + \alpha^2 x_0^2) \dot{\varphi}_0^2 \frac{x^2 - x_0^2}{1 + \alpha^2 x^2} \geq 0 \right\}.$$

To move the motion of the dynamical system (2.5), (2.6) from the initial to the final position, the control parameters  $\dot{x}_0, \dot{\varphi}_0$  must satisfy the conditions

$$\dot{\varphi}_0 \neq 0, \quad \operatorname{sgn} \dot{x}_0 = \operatorname{sgn}(x_* - x_0), \quad \operatorname{sgn} \dot{\varphi}_0 = \operatorname{sgn}(\varphi_* - \varphi_0).$$

We introduce the parameter  $p = |\dot{x}_0|/|\dot{\varphi}_0|$ . Now the description of the set  $\mathbb{X} = \mathbb{X}(p)$  is simplified. As a result, we have

$$\mathbb{X}(p) = \mathbb{R}^+ \text{ under } p \geq x_0 \sqrt{1 + \alpha^2 x_0^2},$$

$$\mathbb{X}(p) = \mathbb{R}^+ / (0, x_1(p)) \text{ under } p < x_0 \sqrt{1 + \alpha^2 x_0^2},$$

where

$$x_1(p) = \sqrt{\frac{x_0^2(1 + \alpha^2 x_0^2) - p^2}{1 + \alpha^2 x_0^2} + \alpha^2 p^2}.$$

The equation (2.5) is converted to

$$x' = \operatorname{sgn}(x_* - x_0) |\dot{\varphi}_0| \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{x^2 - x_0^2}{1 + \alpha^2 x^2}}, \quad x \in \mathbb{X}. \quad (2.7)$$

It is also valid for  $p = 0$ . Using (2.7) and (2.6), we obtain a differential equation for finding the trajectory of a horizontal movement

$$\frac{dx}{d\varphi} = \frac{\operatorname{sgn}(x - x_0)(1 + \alpha^2 x^2)}{\operatorname{sgn}(\varphi_* - \varphi_0)(1 + \alpha^2 x_0^2)} \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{x^2 - x_0^2}{1 + \alpha^2 x^2}}, \quad x \in \mathbb{X}(p).$$

Integrating the differential equation, we find the equation of the trajectory of a horizontal movement

$$\operatorname{sgn}(x_* - x_0) \int_{x_0}^x \frac{(1 + \alpha^2 x_0^2) ds}{(1 + \alpha^2 s^2) \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} = \operatorname{sgn}(\varphi_* - \varphi_0) (\varphi - \varphi_0), \quad x \in \mathbb{X}(p).$$

The trajectory passes through the end point if the condition is true

$$\left| \int_{x_0}^{x_*} \frac{(1 + \alpha^2 x_0^2) ds}{(1 + \alpha^2 s^2) \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} \right| = |\varphi_* - \varphi_0|, \quad x_* \in \mathbb{X}(p).$$

We select the value of the parameter  $p$  to satisfy the condition obtained. For  $0 < x_0 < x_*$ , the required value of the parameter  $p$  is determined by the equation

$$\int_{x_0}^{x_*} \frac{ds}{(1 + \alpha^2 s^2) \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} = \frac{|\varphi_* - \varphi_0|}{1 + \alpha^2 x_0^2}, \quad 0 \leq p < +\infty. \quad (2.8)$$

For  $0 < x_* < x_0$ , the required value of the parameter  $p$  is determined by the equation

$$\int_{x_*}^{x_0} \frac{ds}{(1 + \alpha^2 s^2) \sqrt{p^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} = \frac{|\varphi_* - \varphi_0|}{1 + \alpha^2 x_0^2}, \quad p_{kp} \leq p < +\infty, \quad (2.9)$$

where

$$p_{kp} = \sqrt{\frac{(1 + \alpha^2 x_0^2)(x_0^2 - x_*^2)}{1 + \alpha^2 x_*^2}}.$$

The equation (2.8) has a unique solution  $p = p_*$  under the condition (2.3) and equation (2.9) also has a unique solution  $p = p_*$  if the conditions (2.4) hold.

Suppose that the condition (2.3) holds for  $0 < x_0 < x_*$  and for  $0 < x_* < x_0$  the condition (2.4) is satisfied. Then impulse control  $u_2(t) = \dot{x}_0 \delta(t)$ ,  $u_3(t) = \dot{\varphi}_0 \delta(t)$  ( $t \in \mathbb{R}$ ) bring the dynamic system (1.2), (1.3) to the given final position, if the control parameters are determined by the formulas

$$\dot{x}_0 = |\dot{x}_0| \operatorname{sgn}(x_* - x_0), \quad \dot{\varphi}_0 = |\dot{\varphi}_0| \operatorname{sgn}(\varphi_* - \varphi_0), \quad |\dot{x}_0| = p_* |\dot{\varphi}_0|,$$

where  $p_*$  is the root of the equation (2.8) for  $0 < x_0 < x_*$ , and for  $0 < x_* < x_0$  there is a root of the equation (2.9).

Integrating the differential equation (2.7), we find the arrival time of the motion of the dynamical system (1.2), (1.3) at the end point

$$\tau_2 = \frac{1}{|\dot{\varphi}_0|} \left| \int_{x_0}^{x_*} \frac{ds}{\sqrt{p_*^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} \right|.$$

We synchronize the arrival times of the movements of the dynamical systems (1.1)–(1.3) to the end points.  $\square$

**Theorem 1.** *Let the conditions of Lemma 2 be satisfied. Then the values of the parameters of the impulse controls that move the dynamical system (1.1)–(1.3) from the initial position  $(z_0, x_0, \varphi_0)^\top$ ,  $z_0, x_0 > 0$ , to end position  $(z_*, x_*, \varphi_*)^\top$ ,  $z_* > 0$ ,  $x_* > 0$ ,  $z_* \neq z_0$ ,  $x_* \neq x_0$ ,  $\varphi_* \neq \varphi_0$  are defined by formulas*

$$\begin{aligned} \dot{z}_0 &= \sqrt{2g(z_* - z_0)} \quad \text{for } 0 < z_0 < z_*, \\ \dot{z}_0 &= 0 \quad \text{for } 0 < z_* < z_0, \\ \dot{\varphi}_0 &= \frac{\sqrt{g} \operatorname{sgn}(\varphi_* - \varphi_0)}{\sqrt{2|z_* - z_0|}} \left| \int_{x_0}^{x_*} \frac{ds}{\sqrt{p_*^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} \right|, \\ \dot{x}_0 &= \frac{\sqrt{g} p_*}{\sqrt{2|z_* - z_0|}} \int_{x_0}^{x_*} \frac{ds}{\sqrt{p_*^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}}. \end{aligned}$$

**P r o o f.** Using Lemma 1 and the arrival time  $\tau_1$  of the motion of the dynamical system (1.1) to the end point, we find the value of the control parameter  $\dot{z}_0$ . The synchronization condition  $\tau_1 = \tau_2$  of the arrival times of the motions of dynamical systems (1.1)–(1.3) at end points determines the arrival time  $\tau = \sqrt{2|z_* - z_0|/g}$  of dynamic system movements (1.1)–(1.3) to the end point  $(z_*, x_*, \varphi_*)^\top$  and the equation for the control parameter  $\dot{\varphi}_0$ . From this equation we find

$$|\dot{\varphi}_0| = \frac{\sqrt{g}}{\sqrt{2|z_* - z_0|}} \left| \int_{x_0}^{x_*} \frac{ds}{\sqrt{p_*^2 + (1 + \alpha^2 x_0^2) \frac{s^2 - x_0^2}{1 + \alpha^2 s^2}}} \right|.$$

Using Lemma 2, we find the control parameters  $\dot{\varphi}_0, \dot{x}_0$ . □

### 3. Stabilization of manipulation robot in a final position

When stabilizing the manipulation robot in a small neighborhood of the final position, we use special positional controls, the choice of which turns the final position into a stable equilibrium position of the controlled system. For this purpose we use substitutions for coordinates

$$z = z_* + \hat{z}, \quad x = x_* + \hat{x}, \quad \varphi = \varphi_* + \hat{\varphi}$$

and controls

$$u_1 = \hat{u}_1 + g, \quad u_2 = \hat{u}_2, \quad u_3 = \hat{u}_3(1 + \alpha^2 x_*^2).$$

In a small neighborhood of the final equilibrium, the controlled system (1.1)–(1.3) is replaced by the following controlled system

$$\hat{z}'' = \hat{u}_1, \quad \hat{x}'' = \hat{u}_2, \quad \hat{\varphi}'' = \hat{u}_3. \quad (3.1)$$

We find the stabilizing control using the theory of optimal stabilization for linear systems with quadratic quality criteria. Choosing the quality criterion

$$J_1 = \int_0^{+\infty} (\hat{z}^2(t) + k_1^2 \hat{z}'^2(t) + \hat{u}_1^2(t)) dt, \quad k_1 > 0, \quad (3.2)$$

for the first control of the system (3.1), we find the stabilizing control

$$\hat{u}_1 = -\hat{z} - \sqrt{k_1^2 + 2} \hat{z}'.$$

We also can find stabilizing controls for the second and third equations in (3.1)

$$\hat{u}_2 = -\hat{x} - \sqrt{k_2^2 + 2} \hat{x}', \quad \hat{u}_3 = -\hat{\varphi} - \sqrt{k_3^2 + 2} \hat{\varphi}'$$

using a quality criteria similar to (3.2) with constants  $k_2$  and  $k_3$ , respectively.

### 4. Numerical modeling

In the numerical simulation of the system motions (1.1)–(1.3), the following values of the parameters of the mechanical system were used

$$m_1 = 20, \quad m_2 = 8, \quad J_1 = 12, \quad J_2 = 6, \quad g = 9.8.$$

The start and the end positions are  $z_0 = 0$ ,  $x_0 = 0$ ,  $\varphi_0 = 0$  and  $z_* = 1.4$ ,  $x_* = 0.5$ ,  $\varphi_* = 1.2$ .

We take the controls

$$u_1(t, z) = u_1^{pr}(t) + u_1^{ps}(z), \quad u_2(t, x) = u_2^{pr}(t) + u_2^{ps}(x), \quad u_3(t, \varphi) = u_3^{pr}(t) + u_3^{ps}(\varphi).$$

The program control is defined by formulas

$$u_1^{pr}(t) = \dot{z}_0 \delta(t), \quad u_2^{pr}(t) = \dot{x}_0 \delta(t), \quad u_3^{pr}(t) = \dot{\varphi}_0 \delta(t), \quad t \in \mathbb{R},$$

where the parameters are given by formulas

$$\dot{z}_0 = \sqrt{2gz_*}, \quad \dot{x}_0 = p_* \dot{\varphi}_0, \quad \dot{\varphi}_0 = \sqrt{\frac{g}{2z_*}} \int_0^{x_*} \sqrt{\frac{1 + \alpha^2 s^2}{p_*^2(1 + \alpha^2 s^2) + s^2}} ds.$$

Here  $p = p_*$  is the positive root of the equation

$$\int_0^{x_*} \frac{ds}{\sqrt{(1 + \alpha^2 s^2)((1 + \alpha^2 s^2)p^2 + s^2)}} = \varphi_*.$$

Impulse controls moves the mechanical system into equilibrium, the initial speeds are

$$z'(+0) = \dot{z}_0, \quad x'(+0) = \dot{x}_0, \quad \varphi'(+0) = \dot{\varphi}_0.$$

We also consider software controls in the form of rectangular impulses, which are approximations of ideal impulses

$$u_1^{pr}(t) = \dot{z}_0 \delta_\Delta(t), \quad u_2^{pr}(t) = \dot{x}_0 \delta_\Delta(t), \quad u_3^{pr}(t) = \dot{\varphi}_0 \delta_\Delta(t), \quad t \in \mathbb{R},$$

where

$$\delta_\Delta(t) = 1/\Delta, \quad t \in (0, \Delta), \quad \delta_\Delta(t) = 0, \quad t \in \mathbb{R}/(0, \Delta), \quad \Delta = 0.1.$$

For these controls, the initial velocities of the equilibrium mechanical system are determined by the formulas  $z'(0) = 0$ ,  $x'(0) = 0$ ,  $\varphi'(0) = 0$ .

Positional controls are determined by the following formulas

$$\begin{aligned} u_1^{ps}(z) &= 0, & 0 < z \leq z_* - \epsilon_1, \\ u_1^{ps}(z) &= g - (z - z_*) - \sqrt{k_1^2 + 2z'}, & z > z_* - \epsilon_1, \\ u_2^{ps}(x) &= 0, & 0 < x \leq x_* - \epsilon_2, \\ u_2^{ps}(x) &= -(x - x_*) - \sqrt{k_2^2 + 2x'}, & x > x_* - \epsilon_2, \\ u_3^{ps}(\varphi) &= 0, & 0 < \varphi \leq \varphi_* - \epsilon_3, \\ u_3^{ps}(\varphi) &= -(1 + \alpha^2 x_*^2)((\varphi - \varphi_*) + \sqrt{k_3^2 + 2\varphi'}), & \varphi > \varphi_* - \epsilon_3, \end{aligned}$$

where

$$k_1 = 1, \quad k_2 = 1, \quad k_3 = 1, \quad \epsilon_1 = 0.1, \quad \epsilon_2 = 0.1, \quad \epsilon_3 = 0.1.$$

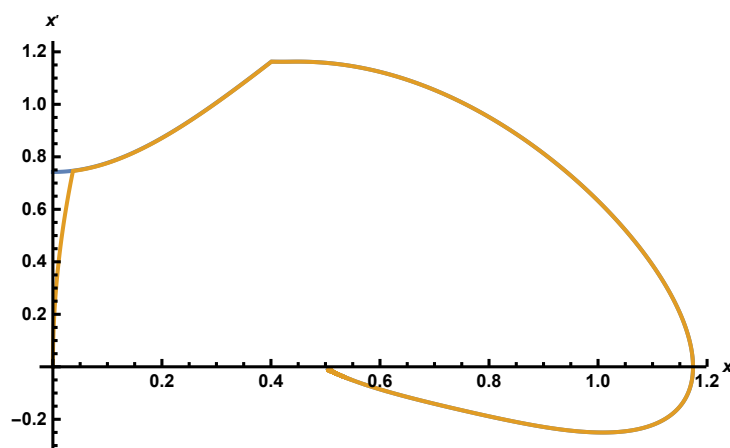
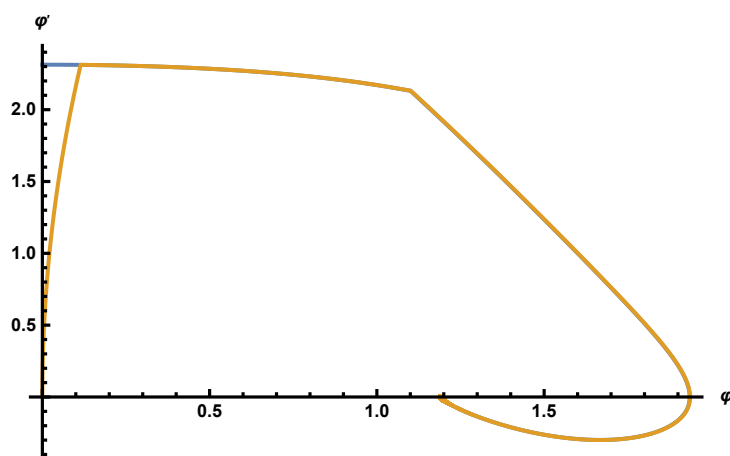
In the final position, the following conditions must be met

$$|z(\tau_f)| < \epsilon, \quad |x(\tau_f)| < \epsilon, \quad |\varphi(\tau_f)| < \epsilon, \quad |z'(\tau_f)| < \epsilon, \quad |x'(\tau_f)| < \epsilon, \quad |\varphi'(\tau_f)| < \epsilon.$$

In the computational experiment, we assumed that  $\epsilon = 0.01$ .

The time movement to the final position is  $\tau_f = 7.5 \text{ sec.}$  for impulse controls and  $\tau_f = 7.51 \text{ sec.}$  for rectangular impulses approximating ideal impulse actions. Projections of phase trajectories on state planes  $(x, x')$  and  $(\varphi, \varphi')$  are shown in Fig. 2 and Fig. 3.

The trajectories corresponding to impulse controls are shown in blue, the trajectories corresponding to the approximations of ideal impulse controls are shown in brown.

Figure 2. State plane  $(x, x')$ .Figure 3. State plane  $(\varphi, \varphi')$ .

## 5. Conclusion

The impulse control is constructed in the work that transfers the manipulator from a given position to its final position. A computational experiment showing the efficiency of the proposed algorithm is presented. The proposed algorithm is simulated in the case when the ideal impulse is approximated by the usual bounded control.

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